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Original article

Restoring Dilapidated Buildings Using Grey Cooperative Games and the fair Distribution of Costs Between the Participating Companies

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Abstract
This paper shows that grey cooperative game theory can help us determine a fair cost between local and international companies to support the problem of restoration, maintenance or even reconstruction of dilapidated housing by using facility location games under uncertainty. Restoration, maintenance, or even reconstruction of dilapidated housing may be a means that must be accomplished before a disaster occurs as pre-emptive planning in order to contribute to better reconstruction and recovery, as well as insuring the lives of residents. In our study, we relied on the old and dilapidated residential quarters in Al-Bayda city. As an example, in our paper, we chose two residential quarter: Red Buildings quarter and White Buildings quarter, as these two quarters are exposed to a disaster that could happen at any moment, God forbid. Some photographs of these dwellings were taken, and they clearly show their deterioration and that they may be on the verge of collapse at any moment (See section 4). Therefore, we will help in restoring and maintaining them through this paper, and two companies will be chosen, one local and the other foreign, for the work for Maintenance and restoration. We will use cooperative grey game theory to help us to restore dilapidated buildings using grey cooperative games and the fair distribution of costs between the participating companies to support this problem using facility location games under uncertainty.
Keywords. Disasters, Dilapidated Housing, Grey Cooperative Games, Grey Numbers.

Introduction

During the past four decades, Libya has been exposed to several natural disasters, such as storms, floods, and earthquakes, but the massive destruction caused by the Mediterranean Hurricane “Daniel” in northern Libya, especially the cities of the eastern region “Cyrenaica,” remains the most violent, severe, and most painful. In February 1963, an earthquake hit the city of Marj (eastern Libya) with a magnitude of about 5.3 on the Richter scale. Epicentre of the earthquake was “the village of Sidi-Dakhil” near Talmitha, causing the death of 243 people and wounding hundreds. It was considered at the time one of the largest natural disasters that Libya had ever witnessed. In its contemporary history. It is noteworthy that Libya has witnessed a number of storms and hurricanes over the past decades, including the “Maximo” storm, which struck the western coast of the country in 1982, followed by the “Seleno” storm near Sirt in January 1995, according to what was monitored by previous study [1].

The Arab Climate Center also monitored Storm “Zeo,” which passed along the Libyan coast in December 2005, then Storm “Rolf” in November 2014, and its impact affected northern Libya. The “Casylda” storm coming from Greece also passed through the coast of Libya in September 2020, but the country only witnessed light to moderate rains at that time in most regions of the northeast, and they were good in the Green Mountain. The following year, some of the coasts of eastern Libya, especially the “Hamama” area, witnessed the “Tornado” hurricane, or as it was known at the time, the mini “funnel” hurricane, but it passed safely over the country. However, Hurricane “Daniel,” which struck Libya in September 2023, and its intensity intensified in the cities of the eastern region, especially the city of Derna, left a “historic tragedy,” according to the description of the Libya weather page concerned with weather news, which confirmed that “Daniel’s” losses were the most severe, destructive, devastating, and painful. Throughout the history of this city, for several reasons, the most important of which is that the city of Derna is located on the banks of the “Wadi Derna,” whose length exceeds 60 kilometers, and the area of its catchment basin is 575 square kilometers. The floods also destroyed two rubble dams (the core was made of compacted clay and the sides were made of stones and rocks) built on the valley's course in the 1970s to protect the city from the floods and torrents to which the city was exposed, which caused loss of life and property. Such disasters may cause the collapse of old buildings, which may collapse naturally due to their age, leaving many residents homeless, and many residents move to neighboring areas, which results in disruption of schooling and rising prices, which may pose a threat to the economic and security situation.

There are many dilapidated buildings that are about to fall in many Libyan cities due to their age. For ten years, governments have not developed a solution to this humanitarian crisis, despite the lack of maintenance and the collapses that claimed the lives of a number of citizens. For example, Balkhair area

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on Omar Al-Mukhtar Street in the Libyan capital, Tripoli, witnessed on the morning of Friday, May 10, 2013, the collapse of a three-story residential building, leaving a number of wounded people who were transported to hospitals to receive treatment. The people of the area stated that the collapsed building dates back to the year it was built. 1918, inhabited by three Libyan families, and it collapsed as a result of the roof of one of the houses falling after the main wall cracked. The capital, Tripoli, also witnessed the collapse of a residential building on Al-Rashid Street, resulting in the death of one of its residents. In a statement regarding the incident of the collapse of some dilapidated buildings within the central municipality of Tripoli, the city's municipal council called on the relevant authorities that the municipality had previously addressed to find urgent solutions regarding the file of buildings that have been on the verge of collapse for decades, and to quickly take measures to address such incidents and limit their recurrence. On the other hand, in the same statement, the Council called on the occupants of those buildings to abide by evacuating them and not to take any action regarding utilizing them as a rental property or the like, due to the lack of safety conditions in them and to preserve the safety of everyone [2]. Hence, our study came to support the solution of a similar problem whose occurrence may lead to a humanitarian catastrophe, as the recurrence of such disasters may cause the collapse of old buildings, which may also collapse naturally due to their age. As a proactive measure, we will offer the restoration and maintenance of these buildings using facility site games in light of uncertainty. Which can help us determine the fair distribution of costs between local and foreign companies to maintain these buildings and ensure the security of their residents.

Basic concepts

The facility location scenario that is crucial to building our model in this section, and in order to provide readers with everything needed to follow this paper, we present some basics from grey cooperative game theory related to the solution ideas are given in the following terms:

Grey number system

Gray theory [3], developed by Professor Deng in 1982, has become a very effective method for solving uncertainty problems under discrete data and incomplete or inaccurate information (information scarcity). Gray theory has been applied to various fields such as prediction, computer graphics system control and decision-making. Here we provide some basic definitions related to the mathematical background related to grey numbers:

The grey number takes an unknown distribution between fixed lower and upper bounds, and is denoted by $\otimes \in [\underline{a}, \overline{a}]$ where $\underline{a}, \overline{a}$ are the lower and upper bounds respectively of $\otimes$. Let $\otimes_1 \in [\underline{a}, \overline{a}], \otimes_2 \in [\underline{b}, \overline{b}], \alpha \in \mathbb{R}_+$ then:

$$\otimes_1 \in [\underline{a}, \overline{a}] + \otimes_2 \in [\underline{b}, \overline{b}] \Leftrightarrow \otimes_1 + \otimes_2 \in [\underline{a} + \underline{b}, \overline{a} + \overline{b}]$$

Numerical multiplication of $\alpha, \otimes$ is defined by $\alpha \otimes \in [\alpha\underline{a}, \alpha\overline{a}]$.

We denote by $\mathcal{G}(\mathbb{R})$ for the set of intervals for grey numbers in $\mathbb{R}$.

Let $\otimes_1, \otimes_2 \in \mathbb{R}$, $|\otimes_1| = \underline{a} - \overline{a}$, $\otimes_1 \in [\underline{a}, \overline{a}]$, $\otimes_2 \in [\underline{b}, \overline{b}], \alpha \in \mathbb{R}_+$ by paragraphs 1 and paragraph 2, we find that $\mathcal{G}(\mathbb{R})$ is a conic section.

In general, the difference between $\otimes_1, \otimes_2$ is defined in the following form:

$\otimes_1 \ominus \otimes_2 = \otimes_1 + (-\otimes_2) \in [\underline{a} - \underline{b}, \overline{a} - \overline{b}]$.

We will use partial subtraction for the subtraction operation defined above if and only when $|\underline{a} - \overline{a}| \geq |\underline{b} - \overline{b}|$ as follows: $\otimes_1 - \otimes_2 = [\underline{a} - \underline{b}, \overline{a} - \overline{b}]$ [4].

Grey cooperative games

In this section, we present an idea of grey cooperative games which are ordered pair $\langle N, w' \rangle$ with a set of players $N = \{1, \dots, n\}$ where $w' = \otimes : 2^N \rightarrow \mathcal{G}(\mathbb{R})$ is the grey reward function, where $w'(\emptyset) = \otimes_\emptyset \in [0,0]$, and the grey reward function $w'(S) = \otimes_S \in [\underline{A_S}, \overline{A_S}]$ denotes To the grey prediction to which the coalition $S \in 2^N$ belongs, where $\underline{A_S}, \overline{A_S}$ represent the minimum and maximum possible profits of the coalition S. Therefore, the grey cooperative game can be considered as a classical cooperative game with grey profits $\otimes$. Gray solutions are useful for solving reward/cost sharing or sharing problems with grey data using cooperative grey gaming as a tool. The basic structure of grey solutions are grey payoff vectors, i.e., vectors whose components belong to $\mathcal{G}(\mathbb{R})$. We denote to $\mathcal{G}(\mathbb{R})^N$ the set of all grey payoff vectors. We symbolize $\mathcal{G}G^N$ for the family of all grey cooperative games. The following example shows a grey game.

Example 1. (Grey glove game): Let $N = \{1, \dots, n\}$ be the set of players that consists of two separate subgroups R, L. The members of group L each have a left glove, and the members of group R each have a right glove. One glove is worth nothing, while the price of the right and left gloves ranges between 5 and 7 dollars. Given $L = \{1\}$ and $R = \{2,3\}$, this situation can be modeled as a three-person grey game, where

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coalitions consisting of players 1 and 2, players 1 and 3, and the grand coalition get part of the value [5 ,7]. The value gained in other cases is [0,0], that is:

$$\otimes_{12} = w'(1,2) = \otimes_{13} = w'(1,3) = \otimes_N = w'(N) \in [5,7].$$

Otherwise, is $\otimes_S = w'(S) \in [0,0]$.

Grey solutions

In this section, we mention a number of grey solutions that are important for our study in this paper. ([5], [6], [7], [8]).

Grey Shapley value

Now, we introduce some theoretical notions from the theory of cooperative grey games. For $w, w_1, w_2 \in IG^N$ and $w', w_1', w_2' \in \mathcal{G}G^N$ we say that $w_1' \in w_1 \leq w_2' \in w_2$ if $w_1'(S) \leq w_2(S)$, where $w_1'(S) \in w_1(S)$ and $w_2'(S) \in w_2(S)$, for each $S \in 2^N$. For $w_1', w_2' \in \mathcal{G}G^N$ and $\alpha \in \mathbb{R}_+$ we define $\langle N, w_1' + w_2' \rangle$ and $\langle N, \alpha w' \rangle$ by $(w_1' + w_2')(S) = w_1'(S) + w_2'(S)$ and $(\alpha w')(S) = \alpha w'(S)$ for each $S \in 2^N$. So, we conclude that $\mathcal{G}G^N$ endowed with "$\leq$" has a cone structure with respect to addition and multiplication with non-negative scalars above. For $w_1', w_2' \in \mathcal{G}G^N$ where $w_1' \in w_1, w_2' \in w_2$ with $|w_1(S)| \geq |w_2(S)|$ for each $S \in 2^N$, $\langle N, w_1' - w_2' \rangle$ is defined by $(w_1' - w_2')(S) = w_1'(S) - w_2'(S) \in w_1(S) - w_2(S)$. We call a game $\langle N, w' \rangle$ grey size monotonic if $\langle N, |w| \rangle$ is monotonic, i.e., $|w|(S) \leq |w|(T)$ for all $S, T \in 2^N$ with $S \subset T$. For further use we denote by $SM\mathcal{G}G^N$ the class of grey size monotonic games with player set N. The grey marginal operators and the grey Shapley value are defined on $SM\mathcal{G}G^N$. Denote by $\prod(N)$ the set of permutations $\sigma : N \to N$ of N. The grey marginal operator $m^\sigma : SM\mathcal{G}G^N \to \mathcal{G}(\mathbb{R})^N$ corresponding to σ, associates with each $w' \in SM\mathcal{G}G^N$ the grey marginal vector $m^\sigma(w')$ of w' with respect to σ defined by

$$m_i^\sigma(w') := w'(P^\sigma(i) \cup \{i\}) - w'\big(P^\sigma(i)\big) \in \left[\underline{A_{P^\sigma(i)\cup\{i\}}} - \underline{A_{P^\sigma(i)}}, \overline{A_{P^\sigma(i)\cup\{i\}}} - \overline{A_{P^\sigma(i)}}\right],$$

for each $i \in N$, where $P^\sigma(i) = \{r \in N \mid \sigma^{-1}(r) < \sigma^{-1}(i)\}$, and $\sigma^{-1}(i)$ denotes the entrance number of player i. For grey size monotonic games $\langle N, w' \rangle, w'(T) - w'(S) \in w(T) - w(S)$ is defined for all $S, T \in 2^N$ with $S \subset T$ since $|w(T)| = |w|(T) \geq |w|(S) = |w(S)|$. We notice that for each $w' \in SM\mathcal{G}G^N$ the grey marginal vectors $m^\sigma(w')$ are defined for each $\sigma \in \prod(N)$, because the monotonicity of $|w|$ implies $\overline{A_{S\cup\{i\}}} - \underline{A_{S\cup\{i\}}} \geq \overline{A_S} - \underline{A_S}$, which can be rewritten as $\overline{A_{S\cup\{i\}}} - \overline{A_S} \geq \underline{A_{S\cup\{i\}}} - \underline{A_S}$.

So, $w'(S \cup \{i\}) - w'(S) \in w(S \cup \{i\}) - w(S)$ is defined for each $S \subset N$ and $i \notin S$. Next, we notice that all the grey marginal vectors of a grey size monotonic game are efficient grey payoff vectors. The grey Shapley value $\Phi' : SM\mathcal{G}G^N \to \mathcal{G}(\mathbb{R})^N$ is defined by

$$\Phi'(w') := \frac{1}{n!}\sum_{\sigma \in \prod(N)} m^\sigma(w') \in \left[\frac{1}{n!}\sum_{\sigma \in \prod(N)} m^\sigma(\underline{A}), \frac{1}{n!}\sum_{\sigma \in \prod(N)} m^\sigma(\overline{A})\right] \quad (1)$$

for each $w' \in SM\mathcal{G}G^N$. We can write the last equation as follows:

$$\Phi_i'(w') := \frac{1}{n!}\sum_{\sigma \in \prod(N)} \left[w'(P^\sigma(i) \cup \{i\}) - w'\big(P^\sigma(i)\big)\right] \in \left[\frac{1}{n!}\sum_{\sigma \in \prod(N)} \underline{A_{P^\sigma(i)\cup\{i\}}} - \underline{A_{P^\sigma(i)}}, \frac{1}{n!}\sum_{\sigma \in \prod(N)} \overline{A_{P^\sigma(i)\cup\{i\}}} - \overline{A_{P^\sigma(i)}}\right] \quad (2)$$

The following example illustrates the calculation of the grey Shapley value.

Example 2. Let $\langle N, w' \rangle$ be a grey cooperative game with $N = \{1,2\}$ set of two players, and let $\otimes_1 = w'(1) \in [5,7], \otimes_N = w'(12) \in [9,12]$, otherwise $\otimes_S = w'(S) \in [0,0]$.

The grey marginal vectors are given in Table 1. where $\sigma : N \to N$.

Table 1. Grey marginal vectors of the cooperative grey game

$\boldsymbol{\sigma}$	$\mathbf{m_1^\sigma(w')}$	$\mathbf{m_2^\sigma(w')}$
$\boldsymbol{\sigma_1 = (1,2)}$	$m_1^{\sigma_1}(w') \in [5,7]$	$m_2^{\sigma_1}(w') \in [4,5]$
$\boldsymbol{\sigma_2 = (2,1)}$	$m_1^{\sigma_2}(w') \in [9,12]$	$m_2^{\sigma_2}(w') \in [0,0]$

The average of two grey marginal vectors is the Shapley grey value for this game, which is given as: $\Phi_i'(w') \in \left(\left[7, \frac{19}{2}\right], \left[2, \frac{5}{2}\right]\right)$.

The grey Banzhaf values

The Banzhaf value arises from the subjective belief that every player is equally possible to join any coalition. On the opposite hand, the Shapley value arises from the idea that for each player, the coalition he joins is equally likely to be of any size which all coalitions of a given size are equally likely [8].

The grey Banzhaf value $\beta : SM\mathcal{G}G^N \to \mathcal{G}(\mathbb{R})^N$, $\forall\, w' \in SM\mathcal{G}G^N$ is defined as

$$\beta(w') = \frac{1}{2^{|N|-1}}\sum_{i \in S}[w'(S) - w'(S \backslash \{i\})] \quad (3)$$

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The $\mathcal{G}CIS$ –value

The CIS-value [9] assigns to every player its individual worth, and distributes the remainder of the worth of the grand coalition N equally among all players [10]. The grey CIS-value assings every player to its individual grey worth, and distributes the remainder of the grey worth of the grand coalition N equally among all players [10]. The $\mathcal{G}$CIS-value $\mathcal{G}CIS: SM\mathcal{G}G^N \to \mathcal{G}(\mathbb{R})^N$ is defined by

$$\mathcal{G}CIS_i(w') = w'(\{i\}) + \frac{1}{2^{|N|}}\left[w'(N) - \sum_{j\in N} w'(\{j\})\right] \quad (4)$$

For more details, see ([13], [14]).

Facility Location Situations

Facility location games were studied by [11]. Moreover, facility location interval games were introduced by [12] in this study. We introduce gray facility location games where we know a set $\mathcal{A}$ of agents, a set $\mathcal{F}$ of facilities, that the cost of opening a gray facility f_i' for each facility $i \in \mathcal{F}$ and the distanced d_{ij} between each pair (i, j) of points in $\mathcal{A} \cup \mathcal{F}$ denotes the cost The grayscale for linking j to i is given by:

$$f_i' \in \left[\underline{f_i'}, \overline{f_i'}\right].\, d_{ij}' \in \left[\underline{d_{ij}'}, \overline{d_{ij}'}\right] \in \mathcal{G}(\mathbb{R}),$$

Distances are assumed to come from a metric space. So, these distances are similar and satisfy the trigonometric inequality. For a set $S \subseteq \mathcal{A}$ of agents, the grey cost of this set is defined as the minimum grey cost of opening a set of facilities and connecting each agent in S to an open facility. More precisely, the grey cost function w' is defined by:

$$w'(S) \in \left[\min_{\mathcal{F}^*\subseteq\mathcal{F}}\left\{\sum_{i\in\mathcal{F}^*}\underline{f_i} + \sum_{j\in S}\min_{i\in\mathcal{F}^*}\underline{d_{ij}}\right\}, \min_{\mathcal{F}^*\subseteq\mathcal{F}}\left\{\sum_{i\in\mathcal{F}^*}\overline{f_i} + \sum_{j\in S}\min_{i\in\mathcal{F}^*}\overline{d_{ij}}\right\}\right] \in \mathcal{G}(\mathbb{R}) \quad (5)$$

The facility location game contains two basic objectives:

1. Determine a viable location for capacity consistent with certain specified facility location rules,
2. Distribution of the total cost among the members of the coalition, where the total cost is dispersed using the ideas of theoretical solutions for the cooperative game, which are compatible with grey solutions.

Now, we present a practical application from the case of the facility location game and the related game using grey data, as we were inspired by the paper presented by [11].

Note: If the numbers of player is two, the relation (5) is written in the following form:

$$w'(S) \in \left[\left\{\sum_{i\in\mathcal{F}^*}\underline{f_i} + \sum_{j\in S}\underline{d_{ij}}\right\}, \left\{\sum_{i\in\mathcal{F}^*}\overline{f_i} + \sum_{j\in S}\overline{d_{ij}}\right\}\right] \in \mathcal{G}(\mathbb{R}) \quad (6)$$

Example 3. Figure 2. Shows a facility location game with 2 players, where player 1 represents the first neighborhood (red buildings), and player 2 represents the second neighborhood (white buildings) in the city of Al-Bayda and 1 facility location f. The value for each coalition is calculated using the relation (6) as follows:

$$w'(\{1\}) \in [7, 9];\; w'(\{2\}) \in [6,8];\; w'(\{1, 2\}) \in [9, 12];$$

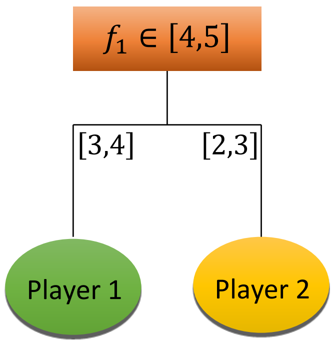

Figure 1. Application of the facility location game with grey data

The gray marginal vectors are given in the following table:

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Table 2. Grey marginal vectors

$\boldsymbol{\sigma}$	$\mathbf{m_1^{\sigma}(w')}$	$\mathbf{m_2^{\sigma}(w')}$
$\boldsymbol{\sigma_1 = (1,2)}$	$m_1^{\sigma_1}(w') \in [7,9]$	$m_2^{\sigma_1}(w') \in [2,3]$
$\boldsymbol{\sigma_2 = (2,1)}$	$m_1^{\sigma_2}(w') \in [3,4]$	$m_2^{\sigma_2}(w') \in [6,8]$

We calculate the Shapley gray value from the table above, so it is

$$\Phi'(w') \in ([5,6.5],[4,5.5]).$$

Now, we calculate the Banzhaf grey value. For the first player we have:

$$\beta_i(w') = \frac{1}{2^{|N|-1}} \sum_{i \in S} [w'(S) - w'(S \backslash \{i\})]$$

$$\beta_1(w') \in \frac{1}{2} \sum_{1 \in S} [w'(S) - w'(S \backslash \{1\})] \in \frac{1}{2}[w'(\{1\}) + w'(\{1,2\}) - w'(\{2\})] \in [5,6.5].$$

In the same way for the other player, we find that:

$$\beta_2(w') \in \frac{1}{2} \sum_{2 \in S} [w'(S) - w'(S \backslash \{2\})] \in \frac{1}{2}[w'(\{2\}) + w'(\{1,2\}) - w'(\{1\})] \in [3.5,5.5].$$

So, the total value of grey Banzhaf is
$\beta(w') \in ([5,6.5],[3.5,5.5])$.
Now, we calculate both the $\mathcal{G}$CIS value as follows:

$$\mathcal{G}CIS_i(w') = w'(\{i\}) + \frac{1}{|N|}\left[w'(N) - \sum_{j \in N} w'(\{j\})\right]$$

$$\mathcal{G}CIS_1(w') \in \; w'(\{1\}) + \frac{1}{2}\left[w'(\{1,2\}) - \sum_{j \in N} w'(\{j\})\right]$$

$$\mathcal{G}CIS_1(w') \in \; w'(\{1\}) + \frac{1}{2}[w'(\{1,2\}) - (w'(\{1\}) + w'(\{2\}))] = [5,6.5].$$

$$\mathcal{G}CIS_2(w') \in \; w'(\{2\}) + \frac{1}{2}[w'(\{1,2\}) - (w'(\{1\}) + w'(\{2\}))] = [4,5.5].$$

The $\mathcal{G}$CIS value is
$\mathcal{G}CIS(w') \in ([5,6.5],[4,5.5])$.
Table 3 shows the results for this application.

Table 3. Grey solutions for our model

Grey Solutions	**Player 1**	**Player 2**
Grey Shapley value	$\in [5,6.5]$	$\in [4,5.5]$
Grey Banzhaf value	$\in [5,6.5]$	$\in [3.5,5.5]$
$\mathcal{G}$CIS-value	$\in [5,6.5]$	$\in [4,5.5]$

Pictures showing dilapidated buildings from the outside and inside

Buildings, like anything on Earth, go through stages of growth and life, from creation to advancement, then ageing, then old age, then disappearance, then the wheel of life turns again, and so on. Cracks are considered one of the most important types of defects that concrete buildings suffer from and are the most common and cause collapses and disasters, despite the development in the field of construction and the attention paid to quality design and good implementation. Here we will show some pictures of the buildings under study, as these pictures show the corrosion of these buildings from the inside and outside, as shown in the following pictures.

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Figure 2. Dilapidated buildings of the neighborhoods under study from inside and outside (Source: [13])

Case study (restoration of residential neighborhoods before the disaster)

We recourse to [14] to help us of this paper. Here we began to consider the restoration of housing in the previously mentioned neighborhoods as a pre-emptive measure before the disaster strikes. Our case study is based on the location of the facility that can be reached even after the disaster, taking into account that a disaster may occur in the city of Al-Bayda, specifically in the old residential neighborhoods such as the building red neighborhood and the building white neighborhood, where their construction was completed in 1962, as shown in the figure 3, which will lead to casualties and a large displacement of residents who will be homeless. Hence, we avoid the displacement of residents to schools and even to neighboring cities, which results in population overcrowding and major economic problems, in addition stop of studies in schools. Therefore, restoring these neighborhoods is a moral and humanitarian duty and is a proactive measure that must occur before the disaster occurs.

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Figure 3. The red buildings and the white buildings, 1962, source [15]

The following table 4 shows the number of housing units, their distribution, as well as the population. We will use the facility location game with grey cooperative games to distribute resources fairly to international organizations or companies involved in the restoration and maintenance of these neighborhoods.

Table 4. Cost of restoration and maintenance of buildings with certain characteristics

Total	**Property and cost of flat**	**Number of flats**	**Name neighborhoods**
[864000,1440000] $ for [720,1152] persons	1 flat =[6000,10000] $ and for [5,8] persons	144 flats (by local company)	Red buildings
[1728000,2880000] $ for [1440,2304] persons	1 flat =[6000,10000] $ and for [5,8] persons	288 flats (by foreign company)	White buildings

In addition, delivery services must be provided to the facility location as well. In our case study, the cost of the service per person ranged between 1,200$ and 1,250$. Table 5 shows the costs of delivery services for companies.

Table 5. Costs of bringing in corporate services

The costs of bringing services of local company	The costs of bringing services of foreign company	Name neighborhoods
[864000,1440000] $ for 144 flats	-	Red buildings
-	[1728000,2880000] $ for 288 flats	White buildings

This costs also includes the construction of service and entertainment facilities.
Figure 4 shows the facility location game with the building red neighborhood (player 1), the building white neighborhood (player 2) in the city of Al-Bayda and two companies, one local and the other foreign.

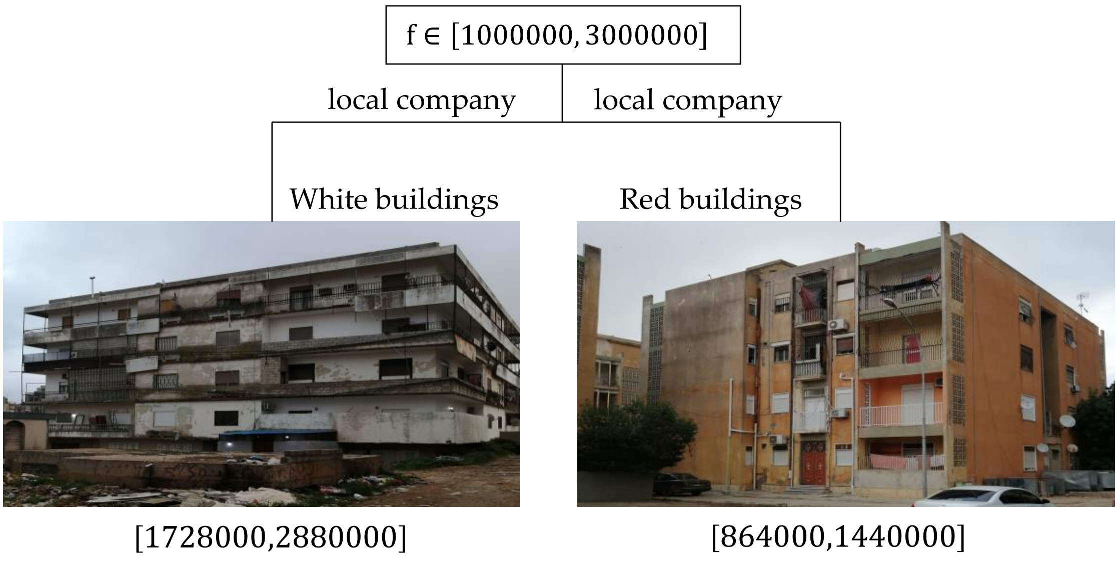

Figure 4. The case study

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The costs for each coalition are calculated using equation (6) as follows:

$$w'(\{1\}) \in [1864000,4440000],$$
$$w'(\{2\}) \in [2728000,5880000],$$
$$w'(\{1,2\}) \in [3592000,7320000].$$

Table 6 shows the marginal vectors of our model, where $\sigma : N \to N$ contains two components of the order (σ_1, σ_2).

Table 6. Marginal vectors of our model

σ	$m_1^{\sigma}(w')$	$m_2^{\sigma}(w')$
$\sigma_1 = (1,2)$	$m_1^{\sigma_1}(w') \in [1864000,4440000]$	$m_2^{\sigma_1}(w') \in [1728000,2880000]$
$\sigma_2 = (2,1)$	$m_1^{\sigma_3}(w') \in [864000,1440000]$	$m_2^{\sigma_3}(w') \in [2728000,5880000]$

The average of the two grey marginal vectors is the grey Shapley value for this game is
$\Phi'(w') \in ([1364000, 2940000], [2228000, 4380000])$
Now, we calculate the grey Banzhaf value for this game. For the first player, we have:

$$\beta_1(w') \in \frac{1}{2^{|N|-1}} \sum_{1 \in S} [w'(S) - w'(S\backslash\{1\})] \in \frac{1}{2}[w'(\{1\}) + w'(\{1,2\}) - w'(\{2\})] \in [1364000, 2940000].$$

For the other player we have:

$$\beta_2(w') \in \frac{1}{2^{|N|-1}} \sum_{2 \in S} [w'(S) - w'(S\backslash\{1\})] \in \frac{1}{2}[w'(\{2\}) + w'(\{1,2\}) - w'(\{1\})] \in [2228000, 4380000].$$

Therefore, the grey Banzhaf value is

$$\beta(w') \in ([1364000, 2940000], [2228000, 4380000])$$

Finally, we calculate the $\mathcal{GCIS}$ value as follows:

$$\mathcal{GCIS}_1(w') \in \ w'(\{1\}) + \frac{1}{N}\left[w'(\{1,2) - \sum_{j \in N} w'(\{j\})\right]$$

$$\mathcal{GCIS}_1(w') \in \ w'(\{1\}) + \frac{1}{2}[w'(\{1,2\}) - (w'(\{1\}) + w'(\{2\}))] = [1364000, 2940000]$$

$$\mathcal{GCIS}_2(w') \in \ w'(\{2\}) + \frac{1}{2}[w'(\{1,2\}) - (w'(\{1\}) + w'(\{2\}))] = [2228000, 4380000].$$

The value of $\mathcal{GCIS}$ is given by
$\mathcal{GCIS}(w') \in ([1364000, 2940000], [2228000, 4380000])$.
Table 7 shows the results of this application.

Table 7. Grey solutions for our model

Grey Solutions	Player 1	Player 2
Grey Shapley value	$\in [1364000, 2940000]$	$\in [2228000, 4380000]$
Grey Banzhaf value	$\in [1364000, 2940000]$	$\in [2228000, 4380000]$
$\boldsymbol{\mathcal{GCIS}}$-value	$\in [1364000, 2940000]$	$\in [2228000, 4380000]$

We note that this result is very satisfactory, as all solutions give us the same results.

Conclusion

The rapid pace of change and scarcity of information after disasters makes it inherently difficult to identify action items. Under these conditions of uncertainty, we use cooperative grey games. The primary goal of the theory is to study ways to maintain cooperation between players under these conditions. The recommendations of this paper identify several key questions and challenges, including the need for efforts to improve the quality and quantity of data on utility site situations, the need for continued efforts to develop post-disaster relief models, and how total costs can be distributed among players in an equitable manner. Through this study, we deal with the problem of restoration and maintenance of dilapidated housing, as we built the cooperative facility location game with the residential neighborhoods referred to previously and showed some grey solution concepts such as the grey Shapley value, the grey Banzhaf value, and the $\mathcal{G}$CSI value. By comparing our results obtained from the grey numbers with the results obtained from other methods, we will see that our results give an idea about the solutions of the periods as well. Because to get a grey solution we have to calculate the interval solutions as well. Since real world models include uncertainty, it is more interesting to work with grey numbers rather than clear solutions. We can develop focused suggestions for future disaster management work to solve problems such as minimization, preparedness, recovery and issues Health and rehousing of displaced populations allows victims to have a private and safe place to return to their normal lives, until permanent housing is rebuilt.

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Conflict of interest. Nil

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المستخلص

توضح هذه الورقة أن نظرية اللعبة التعاونية الرمادية يمكن أن تساعدنا في تحديد التكلفة العادلة بين الشركات المحلية والدولية لدعم مشكلة ترميم أو صيانة أو حتى إعادة بناء المساكن المتداعية باستخدام ألعاب موقع المنشأة في ظل عدم اليقين. قد يكون ترميم أو صيانة أو حتى إعادة بناء المساكن المتداعية وسيلة يجب إنجازها قبل حدوث الكارثة كتخطيط استباقي من أجل المساهمة في إعادة الإعمار والتعافي بشكل أفضل، فضلاً عن تأمين حياة السكان. اعتمدنا في دراستنا على الأحياء السكنية القديمة والمتداعية في مدينة البيضاء. على سبيل المثال، اخترنا في ورقتنا حيين سكنيين: حي المباني الحمراء وحي المباني البيضاء، حيث أن هذين الحيين معرضان لكارثة قد تحدث في أي لحظة، لا قدر الله. وقد تم التقاط بعض الصور لهذه المساكن، وهي تظهر بوضوح تدهورها وأنها قد تكون على وشك الانهيار في أي لحظة (انظر القسم 4). لذلك، سنساعد في ترميمها وصيانتها من خلال هذه الورقة، وسيتم اختيار شركتين، واحدة محلية والأخرى أجنبية، للقيام بأعمال الصيانة والترميم. وسنستخدم نظرية اللعبة التعاونية الرمادية لمساعدتنا في ترميم المباني المتهالكة باستخدام الألعاب التعاونية الرمادية والتوزيع العادل للتكاليف بين الشركات المشاركة لدعم هذه المشكلة باستخدام ألعاب موقع المنشأة في ظل عدم اليقين.

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