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Original article

Study on Some Groups Whose Elements Are Symmetries of Geometric Objects

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ABSTRACT

In this paper discuses some important examples of non-abelian groups whose elements are symmetries of geometric objects, which is called dihedral group D_{2n}, and uses them on some chemical compounds to understand how their atoms vibrate. As we know chemists and mineralogists study dihedral groups in order to classify the structure of molecules and crystals, respectively. Farther more, to study the vibration of atoms of some chemical compounds, we will represent them as geometrical shapes that correspond to the number and arrangement of their atoms in the molecule, with the same angle of binding.

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INTRODUCTION

Sometimes, groups appear to be just sets with some binary operations but in fact most groups are in some sense much more than that. According to definition in the book [1], A is finite set and let symmetry A for the moment denoted the set of one-to-one functions from A onto A. It is easy to see that symmetry A with this composition multiplication forms a group, the symmetry group on set A. One important class of groups in which this “much more” is groups of symmetry, the symmetric group D_n is the set of all symmetries of regular n-gon under composition of transformations. Despite the complexities in the geometric shapes of some compounds, most commonly known as DNA synthesis, the geometric shape of atoms in the molecule becomes easy by following the simplest theories which is mentioned in the book [2].

The geometry of the molecule depends on the number of atoms forming the molecule and the number of free electrons. For instance, the geometrical form of the H2 molecule is a straight part, the reason of those two points can only be connected by straight line. If the molecule consists of three or more atoms, then one of the atoms must be in the middle Symbolized by letter A and the outer atoms symbolized by the letter X and if one of the sides is filled with a pair of free electrons, it is indicated by the letter "E". In this way we can identify the different geometric shapes by following these steps:

1-Draw the Lewis structure.

2- Calculate the number of atoms associated with X and the number of free electron pairs around the middle atom.

3 - Determine the quality of the molecule by letter, for example AX2 or AX2E, and then use the following table-1 to know the geometric shape and then the linkage, see [2]. Note that, we consider and explain the compounds that we need in our paper.

Table 1. The geometric shape of the molecules

Example	The amount of the bonding angle	Describe the geometric shapes	Type of compound	Number Of links	Number of free electrons
$BeCl_2$	180°	Linear	AX_2	2	0
BCl_3	120°	equilateral triangle	AX_3	3	0
NH_4	109.5°	Tetrahedral	AX_4	4	0
PCl_5	72°	Triangle by pyramid	AX_5	5	0
SF_6	60°	Octahedral	AX_6	6	0

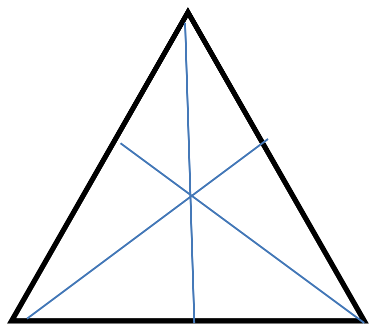
In the book, [3] chemists applied chemistry in group theory by connected the motion of atoms in molecule with the transformations of elements of the group of symmetries D_{2n}, although, the most molecules do not have symmetries, some of them do have symmetries, for instance, if we focus our attention on the iron surrounded by four atoms of hydrogen is similar to square arrangement.

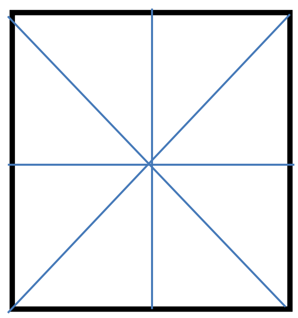
METHODS

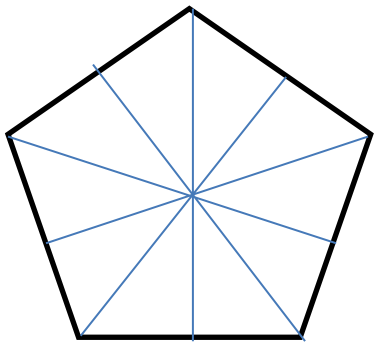
Definition 2.1: [4] Line symmetry: Shapes or patterns are possible contain multiple lines of symmetry, relying on how many times the shape can be folded in half and still remain the same on both sides.
Definition 2.2: [4] Reflective symmetry is when a shape or pattern is reflected in line of symmetry / a mirror line. The reflected shape is going to be exactly the same as the original, the same distance from the mirror line and the same size.
Definition 2.3: [4] Rotational symmetry is when a shape or pattern can be rotated or turned around a central point and remains the same. it is possible to be that a shape has a rotational symmetry of order X; which means that the shape be turned around a central point and remain the same X times.
Definition 2.4: The group D_n is the set of all symmetries of regular n-go n under composition of transformations. In general, it consists of n rotations r (including the identity transformation) and n reflections l.

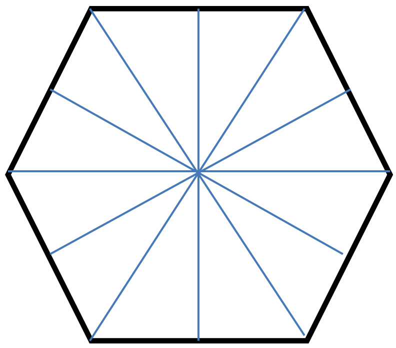
$$D_n = \{1, r, r^2, \dots, r^{n-1}, l, lr, lr^2, \dots, lr^{n-1}\}$$

The order of D_n is equal to 2n.
The order of r is equal to n.
The order of l is equal to 2.
D_n is a nan abelian for n bigger than 2.
The aim of this study is understanding this type of groups and investigating its effects in our life. In both references [5] and [6]: they use this kind of groups we will study reflections and rotations of vertices of polygons for the cases when n=3,4,5,6. As shown below. The easiest way to imagine the transformations identifies the corners of the geometrical shapes and mark them with numbers or symbols.

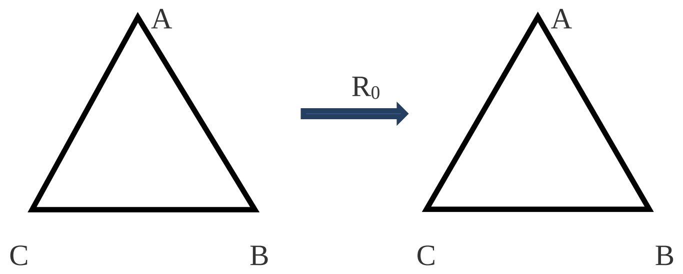




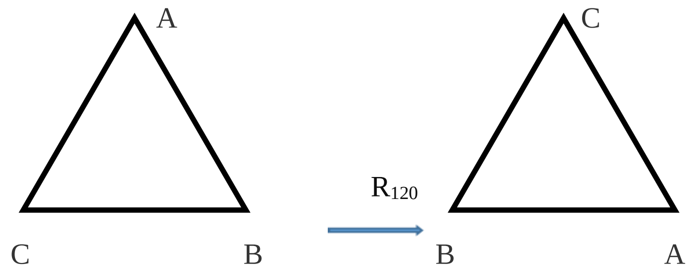




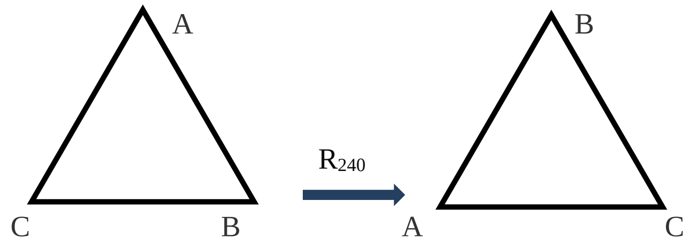
Example (1): Compute the elements of the dihedral group D_3?
This group has order 6(=2(3)) which is found by six symmetries of an equilateral triangle. Here we label the vertices 1, 2, and 3. Rotations of D_3: The triangle is rotated clockwise direction.



Triangle rotated clockwise by 0^0

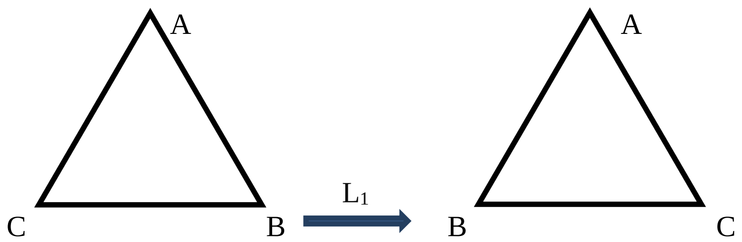


Triangle rotated clockwise by 120^0

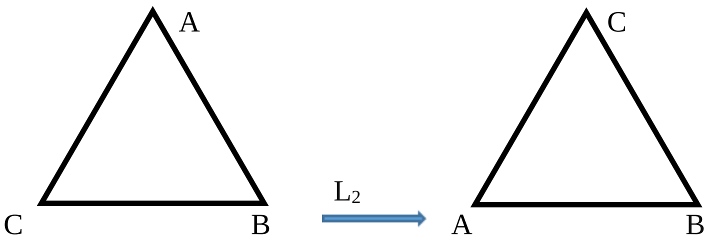


Triangle rotated clockwise by 240^0

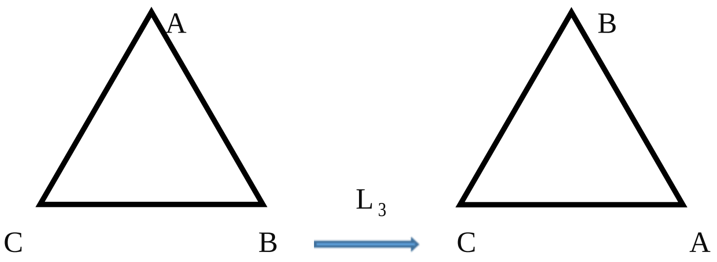
Reflections of D_3:



Triangle reflected across A



Triangle reflected across B



Triangle reflected across C

The multiplication-table of D_3 is:

$(D_{3},.)$	R_0	R_{120}	R_{240}	L_1	L_2	L_3
R_0	R_0	R_{120}	R_{240}	L_1	L_2	L_3
R_{120}	R_{120}	R_{240}	R_0	L_3	L_1	L_2
R_{240}	R_{240}	R_0	R_{120}	L_2	L_3	L_1
L_1	L_1	L_2	L_3	R_0	R_{120}	R_{240}
L_2	L_2	L_3	L_1	R_{240}	R_0	R_{120}
L_3	L_3	L_1	L_2	R_{120}	R_{240}	R_0

Also, for more examples you can find D_4 in [6],[7].

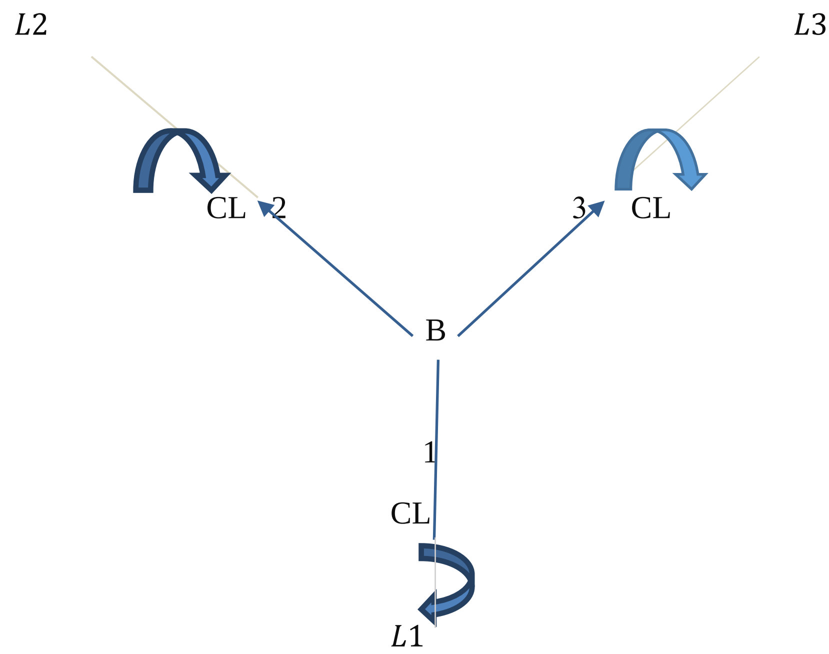
RESULTS AND DISCUSSION

The aim of this study is to understand this type of groups and investigate its Effects in some chemical compounds. We present for example, how to compute the vibrations of an atoms, also understand the groups and their actions.

Example (1): BCl_3

Boron trichloride is a chemical compound of chlorine and boron has the formula BCl3, and is in the form of colorless gas. Where chlorine atoms are bound by the boron atom in the center and the angle between each pair is 120 and then the shape becomes an equilateral triangle.
The discussion of the movement of chlorine atoms in the boron trichloride molecule. We will find and describe the elements of the group:

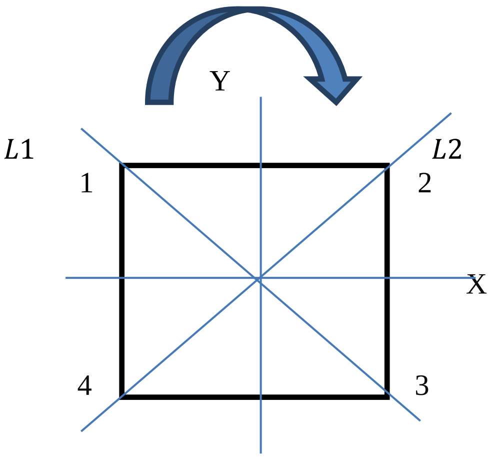
$$D3 = \{1, r, r^2, s, sr, sr^2\}$$

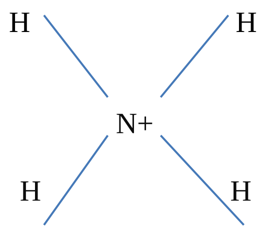


The element 1 is the identity element that keeps each chlorine atom in its place and is expressed in alternation (1).
The element r is the rotation of the chlorine atoms at an angle $\frac{2\pi}{3}$ and expressed by the permutation of the image (123).
The element r^2 is the rotation of the chlorine atoms at an angle $\frac{4\pi}{3}$ and expressed by the permutation of the image (132).
The element s is a reflection of chlorine atoms around the rectum L1 and expressed by alternating images (23).
The element sr is a reflection of chlorine atoms around the rectum L2 and expressed by alternating images (13).
The element sr^2 is a reflection of chlorine atoms around the rectum L3 and expressed by alternating images (12).

Example [2]: NH4+
A mononuclear ion is found in ammonium compounds and can be considered as a product of the ammonia reaction of the Lewis base with the hydrogen ion. This ion has a quadruple shape. Where 4 hydrogen atoms are bound by the nitrogen atom in the center. Also, it gives Tetrahedral shape. We discuss the movement of hydrogen atoms in the ammonium molecule, and study the elements of the group:

$$D4 = \{1, r, r^2, r^3, s, sr, sr^2, sr^3\}$$





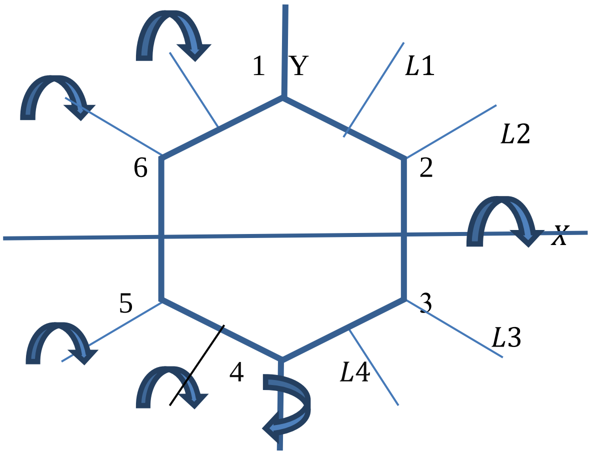
The element 1 is the identity element that keeps each hydrogen atom in its place and is expressed in alternation (1).

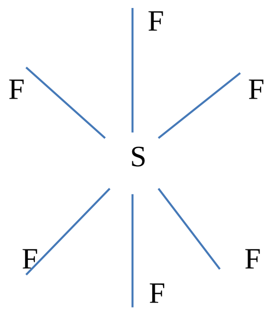
The element r is the rotation of the hydrogen atoms at an angle $\frac{\pi}{2}$ and expressed by the permutation of the image (1234).
The element r^2 is the rotation of the hydrogen atoms at an angle π and expressed by the permutation of the image (13)(24).
The element r^3 is the rotation of the hydrogen atoms at an angle $\frac{3\pi}{2}$ and expressed by the permutation of the image (1432).
The element s is a reflection of hydrogen atoms around the rectum L1 and expressed by alternating images (24).
The element sr is a reflection of hydrogen atoms around the rectum L2 and expressed by alternating images (13).
The element sr^2 is a reflection of hydrogen atoms around the rectum X and expressed by alternating images (14)(23).
The element sr^3 is a reflection of hydrogen atoms around the rectum Y and expressed by alternating images (12)(34).

Example (3): SF_6

Determine vibration of atoms of SF_6 sulfur hexa ,(2000)fluoride, we write the elements of the group D6.

$$D6 = \{1, r, r^2, r^3, r^4, r^5, s, sr, sr^2, sr^3, sr^4, sr^5\}$$





The element 1 is the identity element that keeps each fluorine atom in its place and is expressed in alternation (1).
The element r is the rotation of the fluorine atoms at an angle $\frac{\pi}{3}$ and expressed by the permutation of the image (123456).
The element r^2 is the rotation of the fluorine atoms at an angle $\frac{2\pi}{3}$ and expressed by the permutation of the image (135)(246).
The element r^3 is the rotation of the fluorine atoms at an angle π and expressed by the permutation of the image (14)(25)(36).
The element r^4 is the rotation of the fluorine atoms at an angle $\frac{4\pi}{3}$ and expressed by the permutation of the image (153)(264).
The element r^5 is the rotation of the fluorine atoms at an angle $\frac{8\pi}{3}$ and expressed by the permutation of the image (165432).
The element s is a reflection of the fluorine atoms around the rectum Y and expressed by alternating images (26)(35).
The element sr is a reflection of the fluorine atoms around the rectum L1 and expressed by alternating images (12)(36)(45).
The element sr^2 is a reflection of fluorine atoms around the rectum L2 and expressed by alternating images (13)(46).

The element sr^3 is a reflection of fluorine atoms around the rectum X and expressed by alternating images (14)(23)(56).
The element sr^4 is a reflection of fluorine atoms around the rectum L3 and expressed by alternating images (15)(24).
The element sr^5 is a reflection of fluorine atoms around the rectum L4 and expressed by alternating images (16)(25)(34).

CONCLUSION

In conclusion, the results confirm the effect of the dihedral groups in many sides of science, for instance, in chemistry and nature. The geometric shape of atoms in the molecule becomes easy by following the simplest theories, where the geometry of the molecule depends on the number of atoms forming the molecule and the number of free electrons. For instance, in example BCl_3 chlorine atoms are bound by the boron atom in the center and the angle between each pair is 120 and then the shape becomes an equilateral triangle. For discussion of the movement of chlorine atoms in the boron trichloride molecule. We described the elements of the group D3. In example NH4+, a mononuclear ion. This ion has a quadruple shape. We discussed the movement of hydrogen atoms in the ammonium molecule, and studied the elements of the group D4. Also, in example SF6, to determine vibration of atoms of SF_6 sulfur hexa, (2000) fluoride, we wrote the elements of the group D6.

Conflict of Interest

There are no financial, personal, or professional conflicts of interest to declare.

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دراسة على بعض المجموعات التي تكون عناصرها تماثلات لأجسام هندسية

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المستخلص
يتناول هذا البحث بعض الأمثلة المهمة للمجموعات غير الابدالية التي تكون عناصرها متناظرة من الأجسام الهندسية، والتي تسمى المجموعة ثنائية السطوح D_{2n} ، ويستخدمها على بعض المركبات الكيميائية لفهم كيفية اهتزاز ذراتها. كما نعلم، يقوم الكيميائيون وعلماء المعادن بدراسة المجموعات ثنائية السطوح من أجل تصنيف بنية الجزيئات والبلورات، على التوالي. علاوة على ذلك، لدراسة اهتزاز ذرات بعض المركبات الكيميائية، سنمثلها بأشكال هندسية تتوافق مع عدد ذراتها وترتيبها في الجزيء، وبنفس زاوية الارتباط.
الكلمات الدالة. التماثل الخطي، التماثل الدوراني، التماثل الانعكاسي، المجموعة ثنائية السطوح.