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Original Article

Priority of Applying Stochastic Differential Equations over their Ordinary Counterparts in Accurately Predicting Stock Prices

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ABSTRACT

Background and aims. *Despite the abundance of deterministic models designed to study applied issues in biology, physics, economics, and other fields, which have undergone continuous development and modification to achieve satisfactory future predictions, it is of utmost importance to note that prediction is crucial in the field of economics. It allows sellers and buyers to determine the optimal time for buying and selling. However, as known, market movements are not stable but are subject to numerous and continuous random fluctuations. Therefore, any mathematical model dealing with financial issues must take this into consideration. The aim of this work is to study the solution behavior of one of the most important stochastic economic models, the Black-Scholes model, to demonstrate its superiority over its deterministic counterpart by comparing their results with actual prices.* ***Methods.*** *Prior to studying the stochastic model, its deterministic counterpart was solved, which is represented by an ordinary differential equation. Since calculus of stochastic functions is not very common and differs completely from the known calculus, a series of definitions were provided to explain how to handle them, leading to one of the most important formulas, known as the Itô formula, which was extensively utilized to obtain an explicit solution for the Black-Scholes equation. Additionally, the forms and nature of the solutions were clarified using the MATLAB program.* ***Results.*** *Based on realistic data of a commodity's prices over a four-year period, the necessary coefficients values were calculated, and then substituted into the solution formula for both the deterministic and stochastic models to predict the expected price in both cases after a quarter of a year. The actual price was then revealed, and the results were recorded.* ***Conclusion***. *By comparing the results presented in the table, it is evident that the expected price using the stochastic model is much closer to the actual price than that predicted by the deterministic model, indicating its superiority in prediction accuracy.*

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INTRODUCTION

One of the models useful in random phenomenon, is stochastic processes, among which the evolution of the values of financial (stocks, bonds, …etc) gave a particular interesting stance, and stock exchange take a list of monetary assets [1]. Saving account is asset in a bank, then $S(t)$ is the balance of the saving at time t. Suppose that the bank deposit interest rate is R. Thus, the return $dS(t)/S(t)$ of the saving at time t is Rdt, that is

$$\frac{dS(t)}{S(t)} = Rdt \tag{1}$$

This ordinary differential equation can be solved exactly to give exponential growth in the value of the saving, i.e.

$$S(t) = S_0 e^{Rt} \tag{2}$$

where S_0 is the initial deposit of the saving account at time $t = 0$.

In a risk-free bank, asset prices don't move as money invested, because of the efficient hypothesis market, it is often stated that asset prices must move randomly [2]. Markets are not so steady. The growth rates in the prices of goods and resources and the return rates of many financial assets are affected by many unpredictable factors in the markets [3]. In financial economies, the stochastic modelling has become more and more popular [2]. Mathematics in modern finance can be returned back to Bachelier's dissertation [4] on the theory of speculation in 1900. This pioneering work influences Itô and Samuelson. The underlying asset via a ratio of asset to options were used by the idea of hedging and were recorded in [5], and Thorp used this to invest in the late 1960s. The work of Fischer Black, Myron Scholes and Robert C. Merton in 1973 is the key of revolution in mathematics, by modeling financial markets with stochastic models. The reason that Scholes and Merton were awarded the 1997 Nobel Memorial Prize in Economic Sciences is "for a new method to determine the value of derivatives". In the last few decades, the corporations looked for important tools in terms of financial securities. Options are mainly used to assure assets in order to cover the risks generated in the stock prices changes, as a part of the financial securities. [6, 7].

Since its introduction, financial engineering to price a derivative on equity are widely use the Black– Scholes (B–S) model. Since have been made several generalizations of the initial model, gave premises. Some of these generalizations include stochastic volatility models. [8-13].

The volatility coefficient is constant over the contract time. This has been assumed by the standard Black-Scholes-Merton model. One may find out that the volatility is a function of the exercise time and the strike coefficient, when the parameters of model are calibrated with respect to the market option prices. The form of a convex function is resulted from the dependence curve of the implied volatility of stock prices. This effect is called the "volatility smile" [14].

These days, financial derivatives are becoming increasingly popular, not only as hedging instruments but they are also used more and more frequently for speculative transactions. [15]. Those were mentioned where combined models are considered. Among more recent papers, the Heston-Hull-White models are studied by Grzelak and Oosterlee (2011), Levendis and Maré (2022) and Liu et al. (2023). The Heston-Cox-Ingersoll-Ross model is discussed in Cao et al. (2016) and Mao et al. (2022). [16-20].

The instantaneous change in the price of the asset as the product of two factors, which described by this mathematical model which is often taken to be the geometric Brownian motion. An increase proportional to opportunity cost of capital, i.e., the risk-free interest rate. On the other hand, volatile and unpredictable movements. [21].

There were and still attempts to extend the classical Black-Scholes model (based on geometric Brownian motion) by introducing the randomness into the coefficients which represent, interest rate, drift and volatility or to describe them by SDEs, and then to find explicit or close-form formulas for the hedging strategies and option prices. [22].

Moreover, during the duration of the option, the risk-free interest rate is assumed to be fixed, which refers to the interest rate of the national debts in most cases. Nevertheless, these rates are usually fluctuated rather than fixed. Furthermore, the BS pricing model assumes that the future stock price volatility remains constant, but it is affected by the stock price and other factors (e.g., expiration time of the option), which cannot be kept unchanged. [23].

METHODS

At first, it should be highlighted the related concepts from mathematical view through some definitions, starting with stochastic process

Definition: Stochastic Process is a collection of random variables indexed by t, $\{X(t)|t \geq 0\}$ on the same probability space $(\Omega, \mathcal{F}, P)$. Such that $X(t) = X(t,\omega)$ or $X_t = X_t(\omega)$ with $t \in I$ (I=index set) and for each point $\omega \in \Omega$, the mapping $t \to X(t,\omega)$ is the corresponding sample path. Here, we will interpret t as time and use for I an interval on the real line.

The most important example of stochastic process is the Brownian motion.

Definition: Brownian motion (Wiener Process)

It is a stochastic process $B(t)$ (or $W(t)$) characterized by the following three properties:

1. Normal Increments: $B(t) - B(s)$ has a normal distribution with mean 0 and variance $t - s$. Notice if $s = 0$ that

$B(t) - B(0)$ has normal distribution with mean 0 and variance t.
2. Independence of Increments: $B(t) - B(s)$ is independent of the past.
3. Continuity of Paths: $B(t),\ t > 0$ are continuous functions of t.

These three properties alone define Brownian motion, but they also show why Brownian motion is used to model stock prices. Property 2 shows stock price changes will be independent of past price movements. This was an important assumption we made in our stock price model. An occurrence of Brownian motion from time 0 to T is called a path of the process on the interval $[0, T]$.

Definition: The Variation of $f(t)$ on $[a, b]$ is defined by

$$\text{Variation } (f(t)) = \underbrace{sup}_{all\ partitions} \sum_{i=0}^{n-1} |f(t_{i+1}) - f(t_i)|.$$

If the sup is finite f is said to have bounded variation.

Important Note: Brownian motion does not have bounded variation.

Stieltjes integrals of the form $\int g(t)df(t)$ make sense only when the increment function f has bounded variation and, therefore, $\int_0^t b(s, u(s))dB$ is not well defined as Stieltjes integral [24]. In other words, Brownian motion is not differentiable at any point.

Black– Scholes model

It is the most simple and well-known example model, satisfying the linear stochastic differential equation

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{3}$$

or in the integral form

$$S_t = S_0 + \mu \int_0^t S_u\, du + \sigma \int_0^t S_u\, dW_{u,}\ \ t \in [0, T] \tag{4}$$

the constant μ is called the mean rate of return, and $\sigma > 0$ is called the volatility and shows, in some sense, the degree of risk of that stock. [25].

Since each random variable $X_t = X_t(\omega) = X(t, \omega)$ is a function of 'chance' $\omega \in \Omega$, a stochastic process can be considered as a function of two variables, time $t \in I$ and 'chance' $\omega \in \Omega$; like we do for random variables it is customary to abbreviate the notation and simply write X_t instead of $X_t(\omega)$, but we should keep in mind that the stochastic process depends on 'chance' ω even when such dependence is not explicitly written. This function of t and ω must, of course, when we fix the time t, satisfy the property of being a random variable. (i.e., a measurable function of ω). If we fix the 'chance' ω, we obtain a function of time alone, which is called a trajectory or sample path or realization of the stochastic process. so, a stochastic process can also be considered as a collection of trajectories, one trajectory for each state of the 'chance' ω. The price S_t (abbreviation of $S_t(\omega)$) of a stock at time t for $t \in I = [0, +\infty)$ is an example of a stochastic process. Now ω represents the market scenario, which we may think of as the evolution along time (past, present, and future) of everything that can affect the price of the stock. Obviously, different market scenarios would lead to different price evolutions. For a $t \in I$ fixed, $S_t(\omega)$ is a random variable, therefore a function of 'chance' that associates to each market scenario $\omega \in \Omega$, the corresponding price $S_t(\omega)$ of the stock at time t. For a fixed market scenario $\omega \in \Omega$, the corresponding trajectory $S_t(\omega)$ is a function of time that associates.

As we have already mentioned the second integral of (4) is not well defined as Stieltjes integral. Fortunately, there are other methods could be used to calculate this integral such as Itô integral and Stratonovich integral. Itô integral, however, will be adopted.

Itô Formula

Theorem (The 1-dimensional Itô formula)

Let X_t be an Itô process given by

$$dX_t = udt + vdB.$$

Let $g(t, x) \in C^2([0, \infty) \times R)$ (i.e. g is twice continuously differentiable on $([0, \infty) \times R)$). Then

$$Y_t = g(t, X_t)$$

is again an Itô process, and

$$dY_t = \frac{\partial g}{\partial t}(t, X_t)dt + \frac{\partial g}{\partial x}(t, X_t)dX_t + \frac{1}{2}\frac{\partial^2 g}{\partial x^2}(t, X_t)(dX_t)^2 \tag{5}$$

where $(dX_t)^2 = (dX_t)(dX_t)$ is computed according to the rules

$$dt.dt = dt.dB_t = dB_t.dt = 0, dB_t.dB_t = dt.$$

Proof: [26], pp. 46-48.

Solution of Black-Scholes Equation

Let $Y_t = g(t, S_t) = lnS_t$

According to Itô formula (5), which can be written in more detailed as [27]

$$dY_t = L_0 dt + L_1 dB_t$$

where $L_0(g) = \frac{\partial g}{\partial t} + \mu S_t \frac{\partial g}{\partial s} + \frac{1}{2}(\sigma S_t)^2 \frac{\partial^2 g}{\partial s^2}$ and $L_1 = \sigma S_t \frac{\partial g}{\partial s}$

hence

$$L_0(g) = 0 + \mu S_t \frac{1}{S_t} + \frac{1}{2}(\sigma S_t)^2 \frac{-1}{S_t^2} = \mu - \frac{1}{2}\sigma^2$$

and

$$L_1 = \sigma S_t \left(\frac{1}{S_t}\right) = \sigma$$

Therefore

$$dY_t = d(lnS_t) = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dB_t$$

or

$$\sigma dB_t = d(lnS_t) - \left(\mu - \frac{1}{2}\sigma^2\right) dt$$

Integrating gives

$$\sigma B_t = lnS_t - \left(\mu - \frac{1}{2}\sigma^2\right) t$$

$$lnS_t = \sigma B_t + \left(\mu - \frac{1}{2}\sigma^2\right) t + lnS_0$$

And hence the explicit solution formula for Black-Scholes Equation is

$$S_t = S_0 \, exp\left[\left(\mu - \tfrac{1}{2}\sigma^2\right) t + \sigma B_t\right] \quad [6]$$

Discussion about the Solution

Taking the expectation of the stochastic integral equation,

$$E(S_t) = E[\int_0^t \mu S_\tau d\tau] + cE[\int_0^t S_\tau dB_\tau].$$

Applying the property that the expectation of the Itô integral is equal to zero and interchanging expectation and integration,

$$E(S_t) = \mu \int_0^t E(S_\tau) d\tau.$$

Expressed as an ordinary differential equation,

$$\frac{dE(S_t)}{dt} = \mu E(S_t).$$

Which has the exact solution

$$E(S_t) = E(S_0)e^{\mu t}.$$

hence, the expectation of the Itô solution agrees with the deterministic exponential growth model.

The explicit solution S_t (6), can used with the knowledge about the behavior of B_t to gain the following details

- If $\mu > \frac{1}{2}\sigma^2$ then $S_t \to \infty$ as $t \to \infty$ a.s.

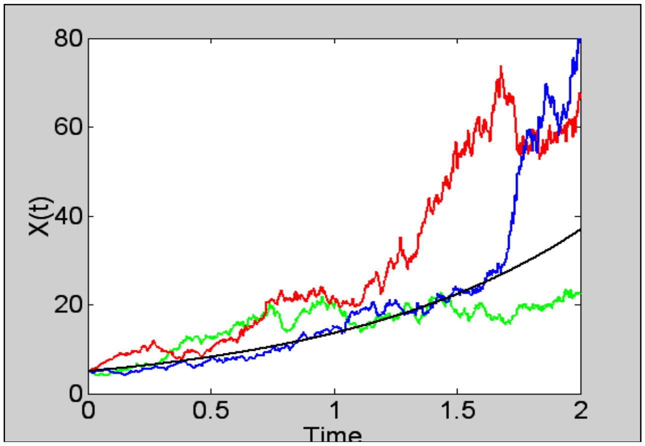

Figure 1. Deterministic Solution & Three Sample Paths of the B-S Model with $\boldsymbol{\mu > \frac{1}{2}\sigma^2}$

- If $\mu < \frac{1}{2}\sigma^2$ then $S_t \to 0$ as $t \to \infty$ a.s.

In this case, contrary of the deterministic model, S_t, with probability one, become extinct even when the mean rate of return (μ) is positive, as long as such value is smaller than $\frac{1}{2}\sigma^2$.

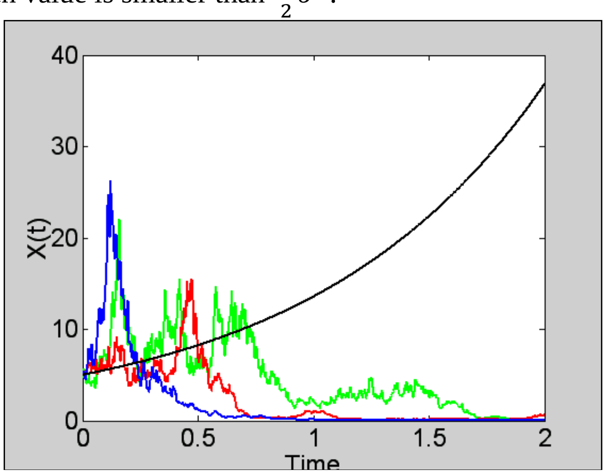

Figure 2. Deterministic Solution & Three Sample Paths of the B-S Model with $\boldsymbol{\mu < \frac{1}{2}\sigma^2}$

- If $\mu = \frac{1}{2}\sigma^2$ then S_t will fluctuate between arbitrary large and arbitrary small values as $t \to \infty$ a.s.

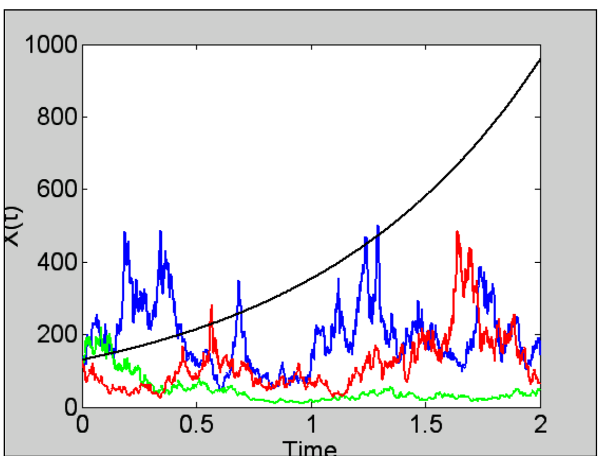

Figure 3. Deterministic Solution & Three Sample Paths of the B-S Model with $\boldsymbol{\mu = \frac{1}{2}\sigma^2}$

RESULTS AND DISCUSSION

Prediction

i. **Using Stochastic Model**

One of the most benefits of financial models and the objective Black-Schole model is to estimate the values of the model parameters through real observed data, and hence use these estimated data to predict future values of stock prices.

Corresponding to the market scenario that has effectively occurred of the price $S(t)$ of a stock (Renault) at the Euronext Paris during the period from the second of January 2012 to 31 December 2015, (using daily closing prices), Braumann [3] used Maximum Likelihood Method to estimate the unknown parameters, and hence to predict the then unknown stock price at 31 March 2016, (i.e., τ = 0.25 years later), the results were as follows:

$S_0 =$ € 27.595 (a stock price on second of January 2012)
$S_n =$ € 92.63 (a stock price on thirty first of December 2015)
$\hat{\mu} = 0.303/year$
$\hat{\sigma}^2 = 0.149/year$

Due to the Markov property, conditional on knowing that $S(t_n) =$ € 92.63, past values are irrelevant for the probability distribution of $S(t_n + \tau)$, the value we want to predict. In this case, it is easier to work with logarithms, using the previous estimated values of parameters, since their real values are not known

$$ln\hat{S}(t_n + \tau) = lnS_n + \hat{\mu}\tau = ln92.63 + 0{,}303 \times 0.25 = 4.6044$$

so that

$$\hat{S}(t_n + \tau) = e^{4.6044} = €99.92$$

This value may not satisfactory enough and close to the realization value, (the true value of the stock that was later observed at 31 March 2016)

$$S(t_n + 0.25) = € \, 87.32$$

ii. **Using Deterministic Model**

Back to Deterministic mode (1)

$$\frac{dS(t)}{S(t)} = Rdt$$

which has an exact solution (2)

$$S(t) = S_0 e^{Rt}$$

substituting for the values of $S_0 = 27.595$, $R = \mu + \frac{\sigma^2}{2} = 0.303 + \frac{0.149}{2} = 0.3775$ and $t = 4.25$, gives

$$S(t) = 27.595e^{(0{,}3775 \times 4.25)} = €137.278$$

Summarize the actual and predicted stock prices estimated by stochastic and deterministic models on 31 March 2016, can be summarized in the following Table

Table 1. The actual and predicted stock prices

Actual stock price	Expected stock price	
	Stochastic model	Deterministic model
€ 87.32	€ 99.92	€ 137.278

Figures (4) and (5) show the similarity between the real and simulated stochastic trajectories, this indicates that the Black-Scholes model does give a reasonable representation of the behavior of stock prices.

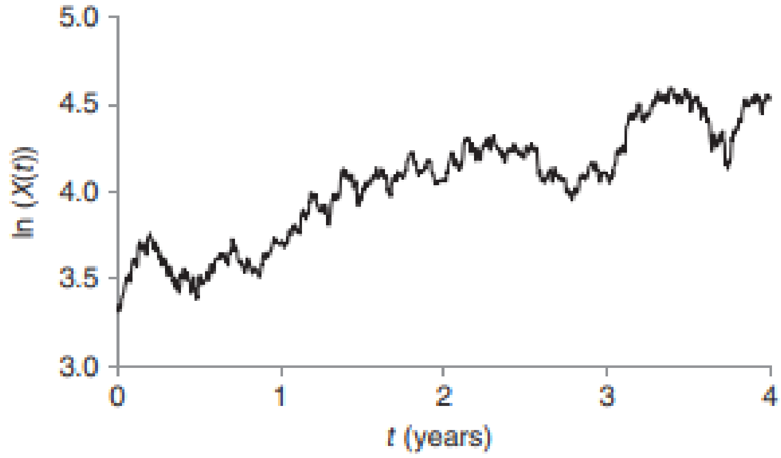

Figure 4: *Observed trajectory of the stochastic process $lnX(t)$, where $X(t)$ is the price in Euros of the Renault stock during the period 2 January 2012 (t = 0) to 31 December 2015 in Euronext Paris. It corresponds to the ω (market scenario) that has effectively occurred. [3],*

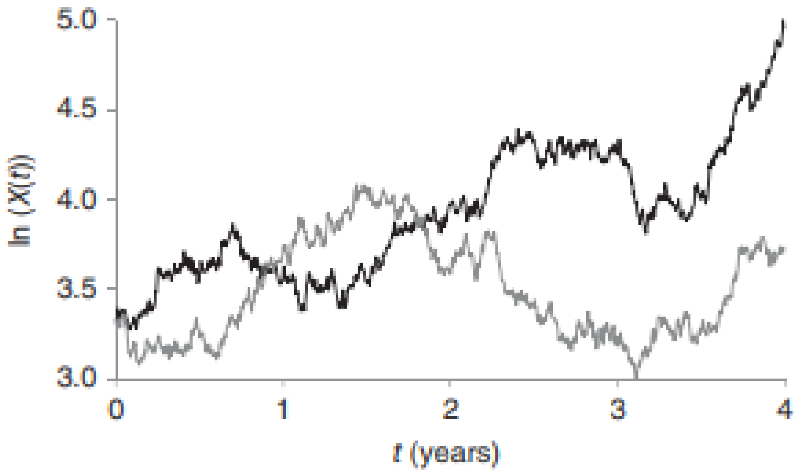

Figure 5: *Two simulated trajectories of the logarithm of Black-Scholes Model [3]*
$\hat{\mu} = 0.303/year$, *and* $\hat{\sigma}^2 = 0.149/year$

Numerical Simulation by MATLAB

To clarify the effect of changing the coefficients values on the behavior of trajectories, MATLAB program is used, the results are shown in figures (6) and (7).

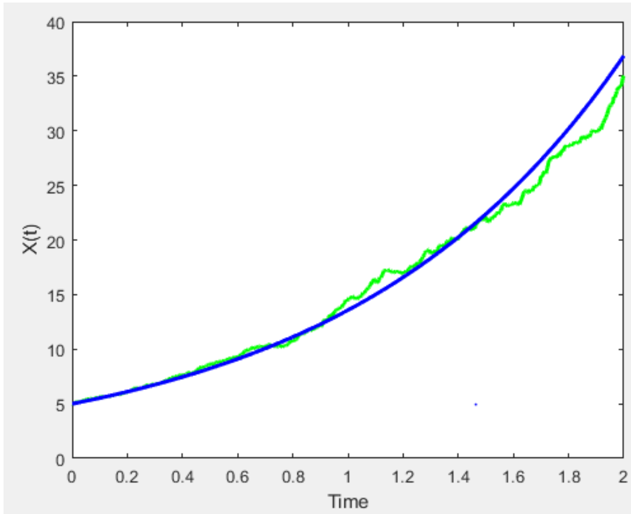

Figure 6: *The greater the average return* (μ) *compared to the volatility* (σ)
$\mu = 0.9$ *and* $\sigma = 0.1$

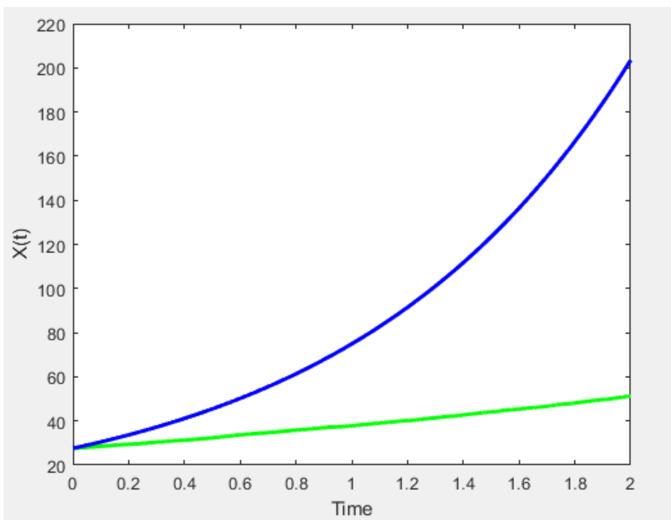

Figure 7: Reducing volatility in the exactly same ω, $\mu = 0.303$ with the exception $\sigma = 0.009$

CONCLUSION

In conclusion, the expected value of stock price obtained by stochastic model is much closer than what can be obtained from deterministic one, Table (1), which indicates the accuracy of the stochastic model and its superiority in prediction over its deterministic Counterpart. The results also showed an effect of the coefficients values: The greater the average return (μ) compared to the volatility (σ) as the random trajectory approached the deterministic trajectory, Figure (6). Moreover, reducing volatility in the exactly same ω, makes the curve less jumpy. Keeping on reducing the volatility until it reaches zero, gives the deterministic model solution, Figure (7).

Conflict of Interest

There are no financial, personal, or professional conflicts of interest to declare.

Appendix

```
%  MATLAB Program for SDE Exponential Model (&deterministic)
% r<c^2/2
```

```
clear
T=2;
k=1000;
dt=T/k;
for j=1:3 % Three Sample Paths
  W(1)=5;
   for i=1:k
     W(i+1)=W(i)+W(i)*dt+2*W(i)*sqrt(dt)*randn;
   end
  plot([0:dt:T],W,'r','LineWidth',2);
  hold on
end
xlabel('Time');
ylabel('X(t)');
  y(1)=W(1);
     for i=1:k % Euler's method for the deterministic model.
        y(i+1)=y(i)+y(i)*dt;
     end
     plot([0:dt:T],y,'b','LineWidth',2)
     hold on
```

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أفضلية تطبيق المعادلات التفاضلية العشوائية على نظيراتها العادية في دقة التنبؤ بأسعار الأسهم

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قسم الرياضيات، كلية العلوم، جامعة عمر المختار، البيضاء، ليبيا

المستخلص

الخلفية والأهداف. على الرغم من وفرة النماذج المحددة والخاصة بدراسة المسائل التطبيقية كما في الأحياء والفيزياء والاقتصاد وغيرها، وما حظيت به من تطوير وتعديل مستمر للحصول على دقة تنبؤ مستقبلية مرضية، وهو أمر في غاية الأهمية، ففي مجال الاقتصاد يعتبر التنبؤ أهم ما يشغل البائع والمشتري ليقدر من خلاله الوقت الأمثل للبيع والشراء. الا أن حركة الأسواق -كما هو معلوم- ليست مستقرة بل تخضع لتقلبات عشوائية عديدة ومستمرة، ولذا فان أي نموذج رياضي يتعامل مع المسائل المالية يجب أن يأخذ ذلك بعين الاعتبار. يهدف ذا العمل الى دراسة سلوك الحل لأحد أهم النماذج العشوائية الاقتصادية وهو نموذج بلاك-سكولس وبيان أفضليته على نظيره المحدد وذلك من خلال مقارنة نتائجهما بالأسعار الفعلية. **طرق الدراسة.** قبل الشروع في دراسة النموذج العشوائي تم حل نظيره المحدد واهو عبارة عن معادلة تفاضلية عادية. نظرا لأن حساب التفاضل والتكامل للدوال العشوائية غير شائع كثيرا ويختلف كليا عن حساب التفاضل والتكامل المعروف أسبق بسلسلة من التعاريف لبيان كيفية معالجته وصولا الى احدى أهم الصيغ وهي صيغة ايتو والتي استخدمت بالتفصيل للحصول على حل صريح لمعادلة بلاك-سكولس كذلك فان أشكال وطبيعة الحلول وضحت من خلال برنامج الماثلات. **النتائج** استنادا الى بيانات واقعية لأسعار احدى السلع خلال فترة أربع سنوات تم حساب قيم المعاملات اللازمة ثم التعويض بها في صيغة حل النموذج المحدد ومن ثم العشوائي للتنبؤ بالسعر المتوقع في الحالتين بعد ربع سنة قادمة، ثم كشف عن السعر الفعلي وتم تدوين النتائج. **الخاتمة.** بمقارنة النتائج الواردة في الجدول يتضح أن السعر المتوقع باستخدام النموذج العشوائي أقرب بكثير الي السعر الفعلي من ذلك المتوقع من خلال النموذج المحدد، مما يعطيه أفضلية في دقة التنبؤ.

الكلمات المفتاحية. المعادلات التفاضلية العشوائية، نموذج بلاك-سكولس, أسعار الأسهم.