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Original article

Comparing the Power of Test of Exponential with Prominent Tests against Alternative with Various Kurtosis

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ABSTRACT

Aims. *In this article, we propose a goodness of fit test for the exponential distribution based on some characterization of the exponential distribution.* ***Methods***. *The test is a weighted integral of the squared distance between the integrated distribution function (IDF) and its estimate. A closed form for the test statistic is derived. The mean and variance of the test statistic are derived under the exponentiality assumption. Also, based on Monte Carlo simulations, the power of the test is evaluated at significance level* $\alpha = 0.05$ *for several sample sizes n=5,10,20, and 50.* ***Results.*** *The proposed test* $T_{1.5}$ *is a good competitor of other known exponentiality test statistics.* ***Conclusion***. *it is recommended to use a proposed test when testing a goodness-of-fit for exponential distribution. Specially, against distributions with kurtosis lower than exponential distribution.*

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INTRODUCTION

The exponential distribution has attracted the attention of statisticians and practitioners from different fields because of its nice mathematical features. Besides the exponential distribution widely used lifetime distribution, it has been exploited to model data from different fields such as physical, medical, biological, and ecology. Testing exponentiality of a sample attracted several researchers. In fact, there exist enormous articles in the literature handling exponentiality tests and their properties. Goodness-of-fit tests until 1986 were synthesized by [1].

The exponentiality tests were classified into four cases: The empirical distribution related tests, correlation based tests, characteristic functions based tests, and chi – square related tests [1].

Prominent tests based on measuring the discrepancy between the empirical distribution function (EDF) and the cumulative distribution function (CDF) are s Kolmogorov-Smirnov (KS), Cramer-von Mises (CM), and Anderson-Darling (AD) tests. The KS test measures the maximum distance between the EDF and the CDF whereas the CM test measure the mean square of this distance and AD test measures a weighted mean square of this deviance. Actually. The AD test is an improvement of CM test. Their test for exponentilitiy based on the empirical Laplace transform [2].

Also, developed a test procedure for the exponential distribution [3], used the same principles as were employed in defining and extending the W- statistic for normality [4,5].

Based on the stabilized probability plot suggest a goodness-of-fit statistic [6], critical values are given of test and investigate its power [7]. Moreover, based on maximum correlations is developed a location and scale free goodness of fit statistic [8]. Finally, a test was developed based on the integrated distribution function [9]. To in provide this test, Let $X_1,...,X_n$ be independent and identically distributed random variables with distribution function F. The integrated distribution function (IDF) is defined by:

$$\psi(t) := E(X-t)^{+} = \int_{t}^{\infty}(1-F(x))dx,$$

where t is any real number and $y^{+} = \min\{0, y\}$. Accordingly, the empirical counter part of the IDF above is

$$\psi_n(t) = \int_{t}^{\infty}(1-F_n(x))dx = \frac{1}{n}\sum_{i=1}^{n}(X_i - t)I_{X_i > t},$$

Where $F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I_{X_i \le x}$ is the empirical distribution function and I_A is the indicator function of event A. proposed test for exponentiality based on a weighted squared integrated distance between the IDF and its estimate [9]. This test is defined by:

$$Klar = \hat{\theta}^3 \int_0^{\infty} (\sqrt{n}\ (\psi_n(t) - \psi(t,\hat{\theta})))^2 dt \qquad (1)$$

The test has the following closed form representation:

$$Klar = \frac{n}{2} - 2\sum_{i=1}^{n} e^{-Y_i} - \frac{1}{3n}\sum_{i=1}^{n}(n-i-1)Y_{(i)}^{3} + \frac{1}{n}\sum_{i<j} Y_{(i)}^{3} Y_{(j)},$$

where $Y_i = \frac{X_i}{\bar{X}_n}$, and $Y_{(i)}$, i=1,2 , ..., n are the ith order statistics of $Y_1, \ldots, Y_n$.

To improve the power of the test in (1), the weight function is defined by:

$w(x) = e^{-a\,x/\theta}$, a>0 [2]. So the modified test is of the form:

$$Klar = (a\hat{\theta})^3 \int_0^{\infty} (\sqrt{n}\ (\psi_n(t) - \psi(t,\hat{\theta})))^2 e^{-at/\theta} dt.$$

A computational form of this weighted test is given by:

$$Klar = \frac{2(3a+2)n}{(2+a)(1+a)^2} - 2a^3\sum_{i=1}^{n}\frac{e^{-(1+a)Y_i}}{(1+a)^2} - \frac{2}{n}\sum_{i=1}^{n} e^{-aY_i} + \frac{2}{n}\sum_{i<j}(a(Y_{(j)} - Y_{(i)}) - 2)e^{-aY_{(i)}}.$$

Let f(x) be a continuous and differentiable density function. Then, it can be shown that

$$\int_t^{\infty} x f(x)dx = (1+t)e^{-t} \ iff\ f(t) = e^{-t}, t > 0 \qquad (2)$$

the left hand side of (2) and its mean is a characterization of the exponential distribution. Based on this characterization we utilize Klar's test approach and propose test statistics based on the characterization given in (2).

Our paper is structured as follows. Firstly, we generalized of the chracterization (2). Secondly, we introduce the proposed test and its properties. Then, based on Monte Carlo simulations, computation of some percentiles, the power of the test and comparisons with prominent tests are presented. Finally, the main conclusions of this study.

Characterization of the exponential distribution

As mentioned in the introduction, the IDF for the exponential distribution, exp(1), is a characterization for the exponential distribution that is

$$\int_t^{\infty} x f(x)dx = (1+t)e^{-t} \ iff\ f(t) = e^{-t}, t > 0$$

Theorem 2.1. Let $f(x)$ be a continuous and differentiable density function on R, if t is a positive real number, then

$$\int_t^{\infty} x^m f(x)dx = ce^{-t}\sum_{j=0}^{m}\frac{t^j}{j!}\ \ iff\ \ f(x) = e^{-x}, \quad (3)$$

for some positive constant c .

Proof.

Let $f(x)=e^{-x}$, then integrating the right hand side (r.h.s.) of (3) by parts m times yields

$$\int_t^{\infty} x^m e^{-x} dx = \Gamma(m+1)e^{-t} \sum_{j=0}^{m} \frac{t^j}{j!} ,$$

and hence the sufficiency condition is proved.

To prove necessity, let $f(x) \geq 0$ be a continuous function on $(0,\infty)$ and assume that (3) is satisfied. Then, differentiating both sides of (3) with respect to t, we obtain

$$-t^m f(t) = ce^{-t}[1+t+\frac{t^2}{2!}+\cdots+\frac{t^{m-1}}{(m-1)!}] - ce^{-t}[1+t+\frac{t^2}{2!}+\cdots+\frac{t^m}{m!}]$$

$$= -c\, e^{-t} \frac{t^m}{m!}.$$

Hence , $f(t) = c\, \dfrac{e^{-t}}{m!}.$

For *f(t)* to be a density function c must be m!.

In particular , for m=1, we have

$$\int_t^{\infty} x\, f(x)dx = (1+t)\, e^{-t} \quad iff \;\; f(x) = e^{-x}.$$

Test statistics

Let $f(x;\theta,\eta) = \theta^{-1} e^{-(\frac{x-\eta}{\theta})}, x > \eta\,, \theta > 0$, and be the exponential density function with parameter θ and η , to be denoted by $\exp(\theta,\eta)$, and let $F(x;\theta,\eta) = 1 - e^{-(\frac{x-\eta}{\theta})}, x \geq \eta\,, \theta > 0$, be the corresponding distribution function. In particular, let

$$f(x) = f(x;\theta) \text{ and } F(x) = F(x;\theta).$$

Assume that $X_1,\ldots,X_n$ be a simple random sample drawn from a population with density function $f(x;\theta)$ and let $\exp(\theta,0)$ be the family of exponential distributions with mean (scale parameter) θ and location parameter 0. We want to test the hypothesis $H_0: f(x,\theta) \in \exp(\theta,0)$ versus $H_1: f(x,\theta) \notin \exp(\theta,0)$. Following by [10], We have

$$\int_{\theta t}^{\infty} \frac{x}{\theta} e^{-x/\theta} dx = \int_t^{\infty} z e^{-z} = (1+t)e^{-t} , \qquad (4)$$

The empirical counter part of the left hand side of the integral in (4) is

$$Y_n(t) \equiv \frac{1}{n}\sum_{i=1}^{n} \frac{X_i}{\theta} I_{X_i > \theta t} = \frac{1}{n}\sum_{i=1}^{n} Z_i I_{Z_i > t} \text{ , where } Z_i = \frac{X_i}{\theta}, \text{ i= 1,2,..,n} .$$

By the strong law of large numbers, $Y_n(t)$ converges to its mean which is equal to:

$$\mu(t) = E[Y_n(t)] \equiv E[Z(t)] = (1+t)e^{-t} \text{ , where } Z(t) = ZI_{Z>t} .$$

To compute the variance of $Y_n(t)$, we have

$$Var[Y_n(t)] = \frac{1}{n^2}\sum_{i=1}^{n} Var[Z_i I_{Z_i>t}] = \frac{Var[Z(t)]}{n} = \frac{E[Z^2(t)] - (E[Z(t)])^2}{n} .$$

$$E[Z^2(t)] = \int_t^{\infty} z^2 e^{-z} dz = (t^2+2t+2)e^{-t} \cdot$$

Therefore,

$$Var[Y_n(t)] = n^{-1}[(t^2+2t+2)e^{-t} - (t+1)^2 e^{-2t}] = n^{-1}[(t+1)^2(1-e^{-t})+1]e^{-t} .$$

If θ is unknown, then we replace $Y_n(t)$ by

$$\hat{Y}_n(t) \equiv \frac{1}{n}\sum_{i=1}^{n}\frac{X_i}{\hat{\theta}_n} I_{X_i > \hat{\theta}_n t} = \frac{1}{n}\sum_{i=1}^{n}\hat{Z}_i I_{\hat{Z}_i > t},$$

where $\hat{Z}_i = X_i / \hat{\theta}$, i=1,2,..,n and $\hat{\theta} = \bar{X}_n = n^{-1}\sum_{i=1}^{n} X_i$ is the maximum likelihood estimator of θ.

A metric measuring the distance between $\hat{Y}_n(t)$ and $\mu_n(t) \equiv E[\hat{Y}_i] = E[\hat{Z}_i I_{\hat{Z}_i > t}]$, such as $n\left(\hat{Y}_n(t) - \mu_n(t)\right)^2$, may be suggested as a test for exponentiality. The dependency of this test statistic on t raises a substantial difficulty. In fact, finding the optimal t that gives the highest power of the test, if exists, is a difficult task. To avoid this problem, we average out over all possible values of t. A suitable weight function that assures the existence of the weighted average has to be chosen. A convenient choice of this weight is a function of form e^{-kt}, for some positive constant k. We have $\mu_n(t) = E[\hat{Y}_n(t)] \to E[Y(t)] = \mu(t)$, thus if $\mu_n(t)$ is replaced by $\mu(t)$ the following test, $T_{n,k}$, is adopted as a goodness-of-fit test for exponentiality

$$T_{n,k} = n\int_0^{\infty}\left(\hat{Y}_n(t) - \mu(t)\right)^2 e^{-kt}dt \qquad (5)$$

$$= n\int_0^{\infty}\left(n^{-1}\sum_{i=1}^{n}\hat{Z}_i I_{\hat{Z}_i > t} - \mu(t)\right)^2 e^{-kt}dt$$

$$= n\int_0^{\infty}\left(n^{-2}\sum_{i,j=1}^{n}\hat{Z}_i\hat{Z}_j I_{\hat{Z}_i\wedge\hat{Z}_j > t} - 2\mu(t)n^{-1}\sum_{i=1}^{n}\hat{Z}_i I_{\hat{Z}_i > t} + \mu^2(t)\right)e^{-kt}dt$$

$$= n\left(n^{-2}\sum_{i,j=1}^{n}\hat{Z}_i\hat{Z}_j\int_0^{\hat{Z}_i\wedge\hat{Z}_j} e^{-kt}dt - 2n^{-1}\sum_{i=1}^{n}\hat{Z}_i\int_0^{\hat{Z}_i}(1+t)e^{-(1+k)t}dt + \int_0^{\infty}(1+t)^2 e^{-(2+k)t}dt\right)$$

$$= \frac{1}{nk}\sum_{i,j=1}^{n}\hat{Z}_i\hat{Z}_j(1 - e^{-(\hat{Z}_i\wedge\hat{Z}_j)}) - \frac{2}{(k+1)^2}\sum_{i=1}^{n}(2 + k - \mathrm{e}^{-(1+k)\hat{Z}_i}(2 + k + (1+k)\hat{Z}_i) + \frac{n(k^2+6k+10)}{(k+2)^3}. \qquad (6)$$

As $n \to \infty$, the process $n^{-1}\sum_{i=1}^{n}\left(\hat{Z}_i I_{\hat{Z}_i > t} - \mu(t)\right)$ converges to a zero mean Gaussian process, $Z^*(t)$, with covariance function $k(s,t) = (t^2 + 2t + 2)\ e^{-t} - (1+t)(1+s)\ e^{-(t+s)}; s < t.$

Thus, $T_{n,k} \to T_k = \int_{-\infty}^{\infty}\left(Z^*(t)\right)^2 e^{-kt}dt$, and

$$E[T_{n,k}] \to E[T_k] = \int_0^{\infty} E\left(Z^*(t)\right)^2 e^{-kt}dt$$

$$= \int_0^{\infty} k(t,t)e^{-kt}dt$$

$$= \int_0^{\infty}(t^2 + 2t + 2)\ e^{-(1+k)t}dt$$

$$= \frac{2(k^2+3k+3)}{(k+1)^3}.$$

A closed form distribution for the test statistic in (5) seems to be hard to derive. Thus, Monte Carlo simulations will be applied to get the percentiles of the test statistic and then the power of the proposed test.

Table 1 displays the $(1-\alpha)$ - quantiles for various values of $\alpha, k,$ and the sample size n. These quantiles are obtained, using Mathematica version 7 software, based on 10,000 samples each of size n from the exponential distribution. We notice the convergence of the quantiles as n increases. Moreover, as one may expect, the quantiles decrease as each of k or n increases.

An approximation to the distribution of $T_{n,k}$ is obtained by usingMINITAB package, version 14.

Table 1. Selective empirical quantiles for $T_{n,k}$ with various values of n and k

N	1-α	K								
		0.25	0.5	1	1.25	1.5	2	2.25	2.5	3
5	0.900	0.799	0.475	0.178	0.114	0.074	0.035	0.025	0.018	0.010
	0.950	0.968	0.574	0.217	0.139	0.092	0.043	0.030	0.022	0.012
	0.990	1.699	0.844	0.295	0.191	0.130	0.062	0.044	0.032	0.017
10	0.900	0.972	0.533	0.179	0.111	0.072	0.033	0.024	0.017	0.010
	0.950	1.231	0.656	0.221	0.138	0.089	0.041	0.029	0.021	0.012
	0.990	2.589	1.120	0.342	0.204	0.132	0.060	0.042	0.031	0.017
20	0.900	1.132	0.582	0.185	0.113	0.072	0.033	0.023	0.017	0.010
	0.950	1.465	0.726	0.231	0.141	0.090	0.041	0.029	0.021	0.012
	0.990	2.589	1.120	0.342	0.204	0.132	0.060	0.042	0.031	0.017
50	0.900	1.243	0.599	0.180	0.108	0.068	0.031	0.022	0.016	0.009
	0.950	1.582	0.751	0.240	0.142	0.089	0.040	0.028	0.020	0.011
	0.990	2.249	1.160	0.322	0.203	0.132	0.061	0.044	0.032	0.018
100	0.900	1.182	0.575	0.171	0.105	0.067	0.031	0.023	0.017	0.010
	0.950	1.562	0.751	0.225	0.135	0.085	0.038	0.028	0.020	0.012
	0.990	2.707	1.286	0.377	0.219	0.134	0.058	0.040	0.029	0.016

The results show that $k = 1.5$ is the value at which the test $T_{n,k}$ has the best power compared to other values of k . Based on these results, we will consider $k = 1.5$ only.

Simulated power

As mentioned in the Introduction, a closed form distribution of the proposed test, under the exponentiality assumption, is not available. So the power of the test will be computed using Monte Carlo simulation. We simulate 10,000 samples of sizes $n = 5,10,20$, and 50, from various alternative distributions. These alternatives with various kurtosis will be considered. The alternative distributions are classified into two classes:

Distributions with low kurtosis ranging from 0-18 and Distributions with moderate and high kurtosis. These kurtosis are computed and provided in Table 2.

Table 2. Classification of alternative distributions according to their kurtosis

No.	Class 1				Class2	
	Distributions with Low Kurtosis (lower than Exponential distribution)		Distributions with Low Kurtosis (Slightly higher than Exponential distribution)		Distributions with Moderate and High Kurtosis	
	Alternatives	Kurtosis	Alternatives	Kurtosis	Alternatives	Kurtosis
1	Gamma(3,1)	5.0	Gamma(0.8,1)	10.5	Gamma(0.4,1)	18
2	Weibull(1.4,1)	4.84	Gamma(1,1)	9.0	Weibull(0.4,1)	290.60
3	Weibull(1.8,1)	3.56	Gamma(1.4,1)	7.29	Log-Normal(0,0.7)	20.79
4	Weibull(2,1)	3.25	Gamma(1.8,1)	6.33	Log-Normal(0,0.8)	34.37
5	Uniform(0,1)	1.8	Gamma(2,1)	6.0	Log-Normal(0,1)	113.94
6	Half-Normal	3.87	Gamma(2.4,1)	5.5	Log-Normal(0,1.5)	10078.3
7	Beta(0.5,0.5)	1.5	Weibull(0.8,1)	15.74	JSHAPE(0.12)	21.57
8	Beta(2,2)	2.14	Log-Normal(0,0.5)	8.90	JSHAPE(0.2)	73.8
9	Power(0.5)	2.4	Chi-Square(1)	15		
10	Power(0.8)	1.90	JSHAPE(0.05)	12.13		
11	Power(1.2)	1.79	LIFR(1)	6.43		
12	Power(1.4)	1.83				
13	Power(2)	2.14				
14	Power(3)	2.91				
15	Power(4)	3.79				
16	LIFR(2)	0.90				
17	LIFR(4)	0.20				
18	LIFR(6)	0.09				
19	LIFR(10)	0.03				

The power of the test is computed at a significance level of $\alpha = 0.05$, for sample size $n = 5,10,20$, and 50 and is compared to a set of prominent tests. There tests are Kolmogorov, Cramer-von Mises and Anderson- Darling. The definitions of the used tests are given below.

i) Kolmogorov- Smirnov (Kol)

The computation from used in simulation is $Kol = sup_x|F_n(x) - F(x)| = \max(D^+, D^-)$, where $D^+ = sup_x\{F_n(x) - F(x)\}$ and $D^- = sup_x\{F(x) - F_n(x)\}$.

Where $F_n(x)$ is the empirical distribution function and F(x) is the cumulative distribution of the exponential distribution with θ being replaced by $\hat{\theta} = \bar{X}$. That is, $F(x) = 1 - e^{-x/\bar{x}}$.

ii) Cramer-Van Mises (CM)

The following computational form is used

$CM = \frac{1}{12n} + \sum_{i=1}^{n}\left[\frac{2i-1}{2n} - F\left(x_{(i)}\right)\right]^2$, where *x(i)*, *i=1,…,n*, are the ith order statistics in the sample.

iii) Anderson-Darling (AD)

The computational form is

$A = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\cdot\left[\ln F\left(x_{(i)}\right) + \ln\left(1 - F\left(x_{(n-i+1)}\right)\right)\right]$, where *F(x)* and *x(i) are* as defined in (ii) above.

Based on 10,000 simulations, using the Mathematica version 7 software, the powers of the proposed test, $T_{1.5}$, and all other tests under consideration are evaluated for samples of sizes 5, 10, 20, and 50 against all classes of alternatives. These powers are reported in Appendix A, Tables A1,A2, and A3. Furthermore, the power of $T_{1.5}$ isplotted along the power of each of other tests for all classes of alternatives under consideration. These plots are displayed in Appendix B.

CONCLUSION

The main conclusions based on the tables and graphs that can be drawn from the simulation results of $\boldsymbol{T_{1.5}}$ for all classes of distributions , all considered sample sizes (**n=5,10,20**,and **50**) and at significance level ($\boldsymbol{\alpha = 0.05}$) are the following :

1) The proposed test $\boldsymbol{T_{1.5}}$proves higher power than Kolmogorov- Smirnove test (KS) for all sample sizes when testing against alternatives with kurtosis lower than that of the exponential distribution. We also note that the proposed test $\boldsymbol{T_{1.5}}$provesequivalent power to KS for most of the alternativeswhen testing against alternatives of kurtosis slightly higher than that of the exponential distribution. Whereas when testing against class II of alternatives, KS test has higher power or equivalent to $\boldsymbol{T_{1.5}}$ **.**

2) Compared to Cramer- Von Mises (CM), $\boldsymbol{T_{1.5}}$proves almost higher power than CM when testing against alternatives with kurtosis lower than that of the exponential distribution, but when testing against alternatives with kurtosis slightly higher, moderate, and high power than kurtosis of exponential CM has higher or equivalent power to $\boldsymbol{T_{1.5}}$.

3) In comparison to Anderson- Darling test (AD), $\boldsymbol{T_{1.5}}$proves higher power than CM for most of the alternativesthat have kurtosis within class I **,** whereas AD shows more power than $T_{1.5}$ when testing against alternatives within class II .

4) In summary , it is recommended to use a proposed test $\boldsymbol{T_{1.5}}$when testing a goodness-of-fit for exponential distribution . Specially , against distributions with low kurtosis

Conflict of interest. Nil

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مقارنة القوة لإختبار أسي مع إختبارات بارزة ضد بدائل مختلفة التفرطح

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الملخص
الخلفية والأهداف. في هذه المقالة ، نقترح اختبار حسن المطابقة للتوزيع الأسي بناءً على بعض الخواص المحددة للتوزيع الأسي. **طُرق الدراسة**. الاختبار هو التكامل المرجح لمربع المسافة بين الدالة التوزيعية التكاملية (IDF) وتقديرها. تم اشتقاق الشكل المغلق للاختبار الإحصائي. تم حساب المتوسط الحسابي و التباين للاختبار المقترح. أيضا استنادا على مونتي كارلو ، يتم تقييم قوة الاختبار عند مستوى دلالة $\alpha = 0.05$ لأحجام عينة مختلفة. **النتائج**. الاختبار المقترح $T_{1.5}$ منافسًا جيدًا مقارنة مع اختبارات أخرى معروفة للتوزيع الأسي. **الخاتمة**. يوصى باستخدام الاختبار المقترح خصوصا ضد البدائل التي لها تفرطح أقل من تفرطح التوزيع الأسي.
الكلمات الدالة. الجودة المطردة للملاءمة ، وقوة الاختبار ، والمحاكاة ، والتوزيعات ذات التفرطح المنخفض ، ووظيفة التوزيع المتكاملة.

Appendix A

Table A1

The simulated (× 100) of the tests under consideration at a significance level of $\alpha = 0.05$ when testing exponentiality against distributions with low kurtosis (lower than exponential distribution)

	Sample size n															
	n=5				n=10				n=20				n=50			
Alternatives	KS	CM	AD	$T_{1.5}$	KS	CM	AD	$T_{1.5}$	KS	CM	AD	$T_{1.5}$	KS	CM	AD	$T_{1.5}$
G (3,1)	22	26	16	28	48	54	45	51	80	88	88	84	*	*	*	*
W (1.4,1)	10	11	07	13	16	19	12	20	29	34	30	33	66	73	76	62
W (1.8,1)	19	23	15	26	40	48	37	47	69	81	79	80	99	*	*	99
W (2,1)	25	30	20	33	52	62	51	62	84	92	92	92	*	*	*	*
U (0,1)	16	19	14	25	28	37	28	47	52	65	61	78	93	98	99	99
H-N	08	08	06	10	11	12	09	15	17	20	16	22	40	47	45	40
B (0.5,0.5)	12	14	21	21	18	23	31	40	34	45	58	69	82	92	98	98
B (2,2)	31	39	29	46	64	78	68	80	93	98	98	99	*	*	*	*
Power(0.5)	49	63	50	71	84	95	91	97	99	*	*	*	*	*	*	*
Power(0.8)	23	30	21	38	47	60	50	69	78	90	89	95	*	*	*	*
Power(1.2)	11	12	10	17	18	21	17	30	31	40	37	54	75	86	86	90
Power(1.4)	08	09	10	13	12	14	12	21	18	23	23	37	50	61	66	71
Power(2)	07	08	20	09	10	12	26	14	15	18	40	25	36	47	76	52
Power(3)	16	20	47	16	33	36	65	31	57	63	87	59	95	96	*	93
Power(4)	30	34	66	28	59	62	87	52	87	89	98	84	*	*	*	*
LIFR(2)	09	10	07	12	14	16	11	18	23	28	24	30	56	64	61	55
LIFR(4)	11	13	08	15	20	23	15	24	33	41	36	42	74	83	81	76
LIFR(6)	12	14	09	17	23	26	18	28	40	49	43	50	82	90	90	85
LIFR(10)	14	17	11	19	27	32	23	33	48	58	52	58	90	95	95	92

An asterisk denotes power 100%

Table A2

The simulated (× 100) of the tests under consideration at a significance level of $\alpha = 0.05$ when testing exponentiality against distributions with low kurtosis (slightly higher than exponential distribution)

	Sample size n															
	n=5				n=10				n=20				n=50			
Alternatives	KS	CM	AD	$T_{1.5}$	KS	CM	AD	$T_{1.5}$	KS	CM	AD	$T_{1.5}$	KS	CM	AD	$T_{1.5}$
G (0.8,1)	06	06	10	05	07	07	10	06	08	09	14	07	17	18	25	10
G (1,1)	05	05	05	05	05	05	05	05	05	05	05	05	05	05	05	03
G (1.4,1)	07	07	05	08	09	10	06	10	13	15	12	14	28	32	32	23
G (1.8,1)	10	11	07	13	17	20	13	19	29	35	32	32	69	77	80	63
G (2,1)	12	14	08	16	22	25	18	24	39	47	45	43	83	90	92	79
G (2.4,1)	16	18	11	21	32	37	28	35	59	68	67	63	97	99	99	96
W (0.8,1)	07	08	13	05	11	12	17	08	17	19	26	13	38	43	52	25
L-N (0,0.5)	36	40	29	41	75	81	73	73	98	99	99	98	*	*	*	*
Chis (1)	13	15	30	10	26	38	46	20	46	52	70	39	86	89	97	75
JSHAPE(0.05)	05	05	06	05	05	05	05	05	06	05	07	05	06	06	06	03
LIFR(1)	08	08	05	10	10	11	08	13	15	18	13	19	35	40	38	33

Table A3

The simulated (× 100) of the tests under consideration at a significance level of $\alpha = 0.05$ when testing exponentiality against distributions with moderate and high kurtosis

	Sample size n															
	n=5				n=10				n=20				n=50			
Alternatives	KS	CM	AD	$T_{1.5}$	KS	CM	AD	$T_{1.5}$	KS	CM	AD	$T_{1.5}$	KS	CM	AD	$T_{1.5}$
G (0.4,1)	20	23	47	17	42	46	67	32	71	75	90	61	98	99	*	94
W (0.4,1)	42	47	69	36	78	81	92	68	97	98	99	95	*	*	*	*
L-N (0,0.7)	14	17	10	17	29	33	24	28	55	61	60	51	97	97	99	89
L-N (0,0.8)	10	11	07	12	17	20	13	16	30	33	33	27	72	75	85	55
L-N (0,1)	07	08	05	07	10	10	08	08	13	15	13	09	25	29	35	16
L-N (0,1.5)	16	16	21	11	33	35	37	24	57	61	62	46	91	93	94	82
JSHAPE(0.12)	05	05	07	05	06	07	07	05	08	07	09	05	10	11	12	05
JSHAPE(0.2)	06	07	08	05	08	09	09	06	11	12	14	07	20	22	23	11

Appendix B

Power plots of the proposed test $T_{1.5}$ *along with that of all tests under considerations for all considered classes of alternatives and sample sizes .*

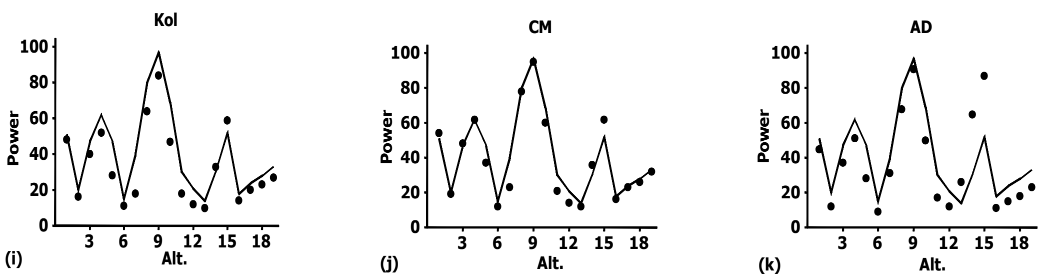

Figure B1 . *Power Plot of the proposed test* $T_{1.5}$ *(continuous line)and the tests* Kol *.*CM *.and AD . respectively . for* $n = 10$ *and* $\alpha = 0.05$ *against distributions with low kurtosis (lower than exponential distribution)*

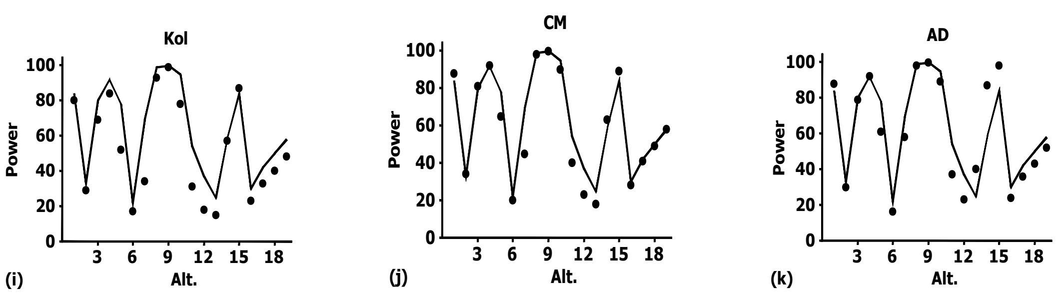

Figure B2 . *Power Plot of the proposed test* $T_{1.5}$ *(continuous line)and the tests* Kol *.*CM *.and AD . respectively . for* $n = 20$ *and* $\alpha = 0.05$ *against distributions with low kurtosis (lower than exponential distribution)*

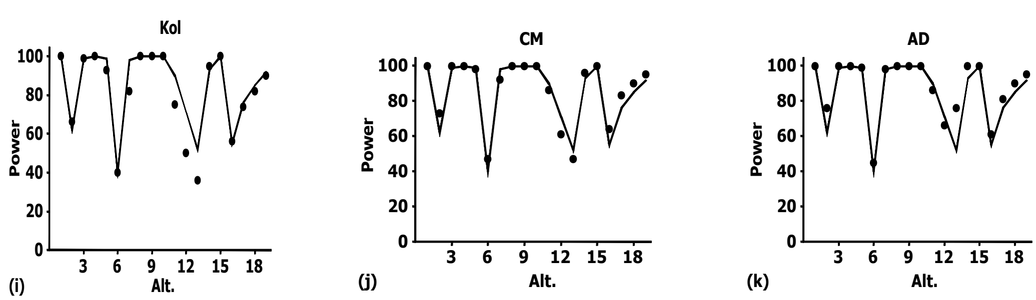

Figure B3 . *Power Plot of the proposed test* $T_{1.5}$ *(continuous line)and the tests* Kol *.*CM *.and AD . respectively . for* $n = 50$ *and* $\alpha = 0.05$ *against distributions with low kurtosis (lower than exponential distribution)*

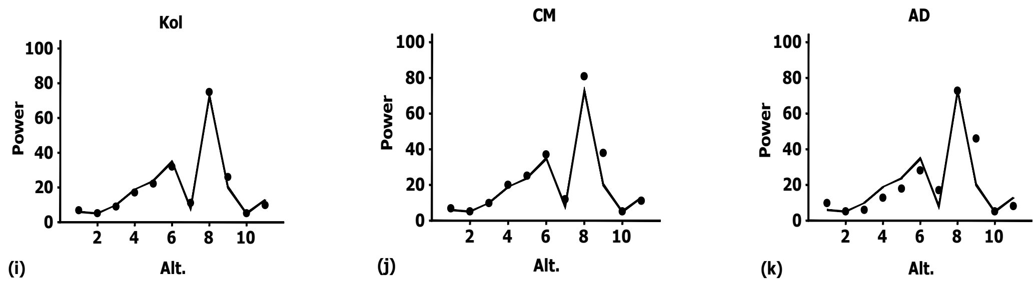

Figure B4 . *Power Plot of the proposed test* $T_{1.5}$ *(continuous line)and the tests* Kol *.*CM *.and AD . respectively . for* $n = 10$ *and* $\alpha = 0.05$ *against distributions with low kurtosis (slightly higher than exponential distribution)*

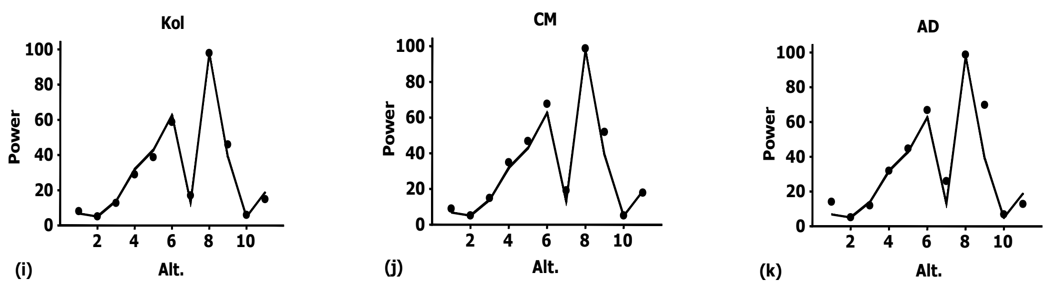

Figure B5 . *Power Plot of the proposed test* $T_{1.5}$ *(continuous line)and the tests* Kol *.*CM *.and AD . respectively . for* $n = 20$ *and* $\alpha = 0.05$ *against distributions with low kurtosis (slightly higher than exponential distribution)*

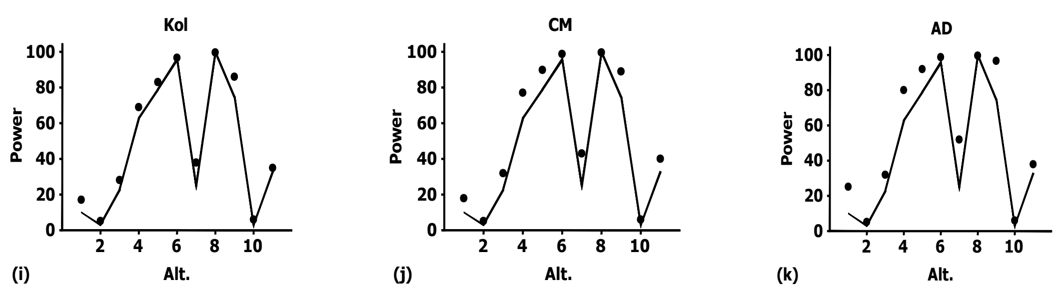

Figure B6 . *Power Plot of the proposed test* $T_{1.5}$ *(continuous line)and the tests* Kol *.*CM *.and AD . respectively . for* $n = 50$ *and* $\alpha = 0.05$ *against distributions with low kurtosis (slightly higher than exponential distribution)*

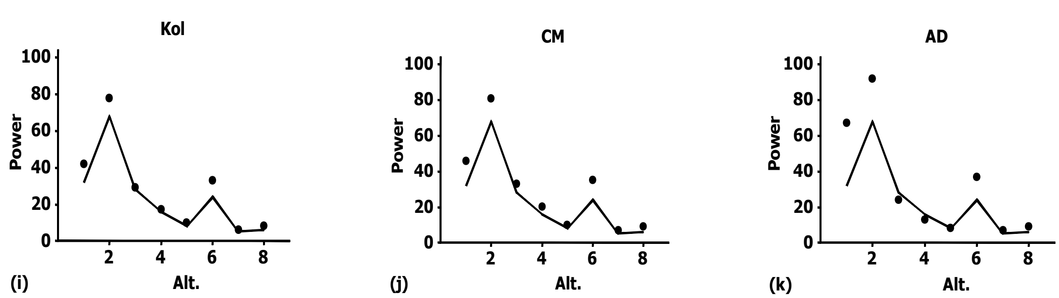

Figure B7 . *Power Plot of the proposed test* $T_{1.5}$ *(continuous line)and the tests* Kol *.*CM *.and AD . respectively . for* $n = 10$ *and* $\alpha = 0.05$ *against distributions with moderate and high kurtosis*

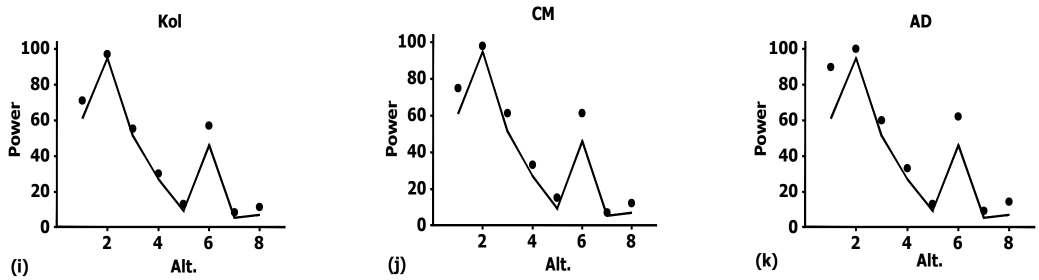

Figure B8 . *Power Plot of the proposed test* $T_{1.5}$ *(continuous line)and the tests* Kol *.*CM *.and AD . respectively . for* $n = 20$ *and* $\alpha = 0.05$ *against distributions with moderate and high kurtosis*

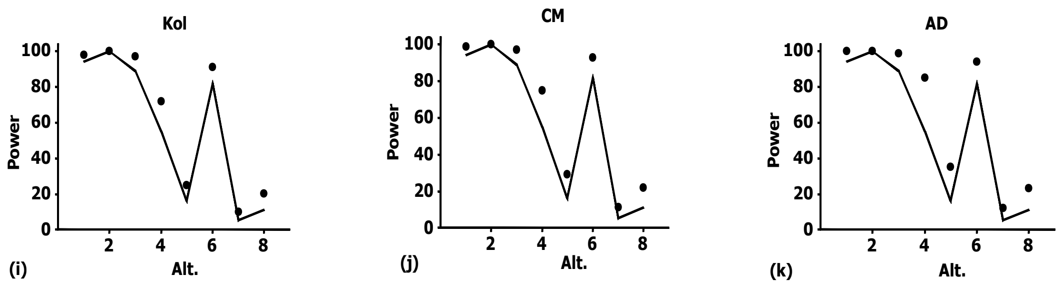

Figure B9 . *Power Plot of the proposed test* $T_{1.5}$ *(continuous line)and the tests* Kol *.*CM *.and AD . respectively . for* $n = 50$ *and* $\alpha = 0.05$ *against distributions with moderate and high kurtosis*