Alqalam Journal of Medical and Applied Sciences. 2025;8(1):73-84
https://doi.org/10.54361/ajmas.258112

Original article

Reliability Analysis of Two-Unit Cold Standby and Warm Standby Outdoor Electric Power Systems in Stable Weather

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Abstract
In this paper presents the reliability analysis two mathematical models representing electric power systems operating in fluctuating outdoor weather (i.e., normal and stormy weather) and compared between two models. Model I deal the reliability analysis of a single-server two-unit cold standby; Model II deals the reliability analysis of a single-server two-unit warm standby. for two systems with two different modes [normal, total failure]. System failure occurs when both the units fail totally. The failure rate and failed repair rate of a unit are constants. Laplace transforms of the various state probabilities have been derived and then reliability is obtained by the inversion process. Moreover, an important parameter of reliability, i.e., MTTF (mean time to failure), the variance transition of time to failure of the system, system availability and steady-state availability are derived. The failure times of operating/spare units and repair time of failed units are exponential distributed. Certain important result has compared between two systems
Keywords. Cold standby, Warm standby, Reliability, Men time to system failure.

Introduction
In many previous scientific papers such a case was addressed with certain different for example in the study of the extent of the effect of the rate of change of weather from moderate to unmoderated on system with one operating unite [1], and another backup unite was addressed also in the study of the comparison of reliability and the availability between four systems with warm standby components and standby switching failures [2]. Another study has rebelled the model and analysis of power system reliability evaluation considering weather change [3], also study have addressed the reliability evaluation of electric power systems in alternating environment [5].
In this research, a comparison is presented between two models representing electrical power systems operating in fluctuating outdoor weather (i.e., normal and stormy weather), where the first model represents two units one of which is working and the other cold standby and the second model represents two units one of which is working and the other worm standby and both systems have two different patterns [normal total failure]. The system breaks down when it is exposed to total failure by deriving the different probabilistic Laplace transform of the two models. Moreover, an important parameter of reliability, i.e., MTTF (mean time to failure), the variance transition of time to failure of the system, system availability and steady-state availability are derived. The failure times of operating/spare units and repair time of failed units are exponential distributed. Certain important result has compared between two systems.

Assumptions
The following assumptions are associate d with models I and II:
1. The system ceases to function when both the units are non-operative,
2. the system operates in changing weather (i.e., normal and stormy weather),
3. As soon as the operating unit fails, it is replaced at a certain constant rate by the standby,
4. At time t=0, one unit is switched into operation and the other one is in cold (or worm) standby mode,
5. Units are identical and statistically independent,
6. Unit failure rate is constant,
7. A repaired unit is as good as 'new'(this assumption is applicable to model I and II).

Notation
Here is the symbols and parameters are associated with both models:

$OU[OU']$ when t=0 one unit is switched into operation and the other unit is in cold [or worm] standby mode,

$iU\ [iU']$ i^{th} up state of the system in normal [stormy] weather (i units are non-operative); i=0,1,2,

$iD\ [iD']$ i^{th} down state of the system in normal [stormy] weather (i units are non-operative); i=1,2,

***Received**: 18-11-2024 - **Accepted**: 06-01-2025 - **Published**: 12-01-2025*

λ $[\lambda']$	Unit constant failure rate operative unit in normal [stormy] weather,
δ $[\delta']$	Unit constant failure rate worm standby unit in normal [stormy] weather,
μ $[\mu']$	Unit constant switching-in rate from standby mode to on-line mode in normal [stormy] weather,
α $[\gamma]$	Weather constant transition rate from normal to stormy [stormy to normal] state,
β $[\beta']$	Unit constant repair rate from stat in normal [stormy] weather,
s	Laplace transform variable,
$P_i(t)$	Probability that the system is in state I at time t.

Analysis
Model I

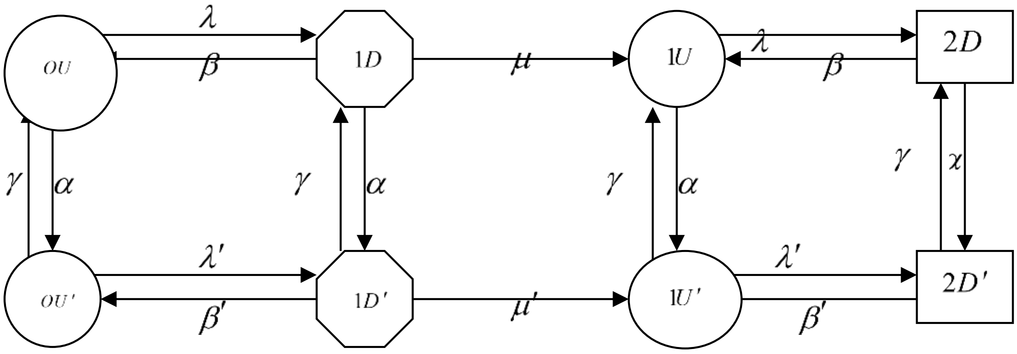

Figure 1. State space diagram for Model I

Using the transitions of the Markov process to the up states of the system. Let $P_{iU[iU']}(t), i=0,1$ *and* $P_{iD[iD']}(t),\ i=1,2$ be the probability that the system is in state $iU[iU'], i=0,1$ *and* $iD[iD'], i=1,2$ at time t. The infinitesimal generator of the Markov process is given below

$$Q_{1}=\begin{bmatrix} -X_4 & \beta & 0 & 0 & \gamma & 0 & 0 & 0 \\ \lambda & -Z_1 & 0 & 0 & 0 & \gamma & 0 & 0 \\ 0 & \mu & -X_4 & \beta & 0 & 0 & \gamma & 0 \\ 0 & 0 & \lambda & -X_5 & 0 & 0 & 0 & \gamma \\ \alpha & 0 & 0 & 0 & -Y_4 & \beta' & 0 & 0 \\ 0 & \alpha & 0 & 0 & \lambda' & -Z_2 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & \mu' & -Y_4 & \beta' \\ 0 & 0 & 0 & \alpha & 0 & 0 & \lambda' & -Y_5 \end{bmatrix}$$

$$X_4=(\alpha+\lambda),\ X_5=(\beta+\alpha),\ Y_4=(\lambda'+\gamma)\ and\ Y_5=(\beta'+\gamma) \quad (1)$$
$$Z_1=(\beta+\mu+\alpha)\ and\ Z_2=(\beta'+\mu'+\gamma)$$

We assume that initially both the units are operable and obtain the measures of system performance.

System reliability

The system reliability R(t) is the probability of failure-free operation of the system in (0, t]. To derive an expression for the reliability of the system, we restrict the transitions of the Markov process to the up states, viz. $iU[iU'],\quad i=o,1$.Using the infinitesimal generator given in (1), pertaining to these states and standard probabilistic arguments, we derive the following differential equations.

Received: 18-11-2024 - **Accepted**: 06-01-2025 - **Published**: 12-01-2025

$$\frac{dp_{Ou}}{dt} = -(\lambda+\alpha)\ p_{Ou}(t) + \gamma\, p_{ou'}(t)\ ,$$
$$\frac{dp_{1U}}{dt} = -(\lambda+\alpha)\ p_{1U}(t) + \gamma\, p_{1U'}(t)\ ,$$
$$\frac{dP_{0U'}}{dt} = -(\lambda'+\gamma)\ P_{0U'}(t) + \alpha\ p_{0U}(t)\ ,$$
$$\frac{dp_{1U'}}{dt} = -(\lambda'+\gamma)\ p_{1U'}(t) + \alpha\ p_{1U}(t)\ .$$
(2-.5)

Let L_i *(s)* be the Laplace transform of $p_{iu[iu']}(t), i=0,1$.Taking Laplace transform on both the sides of the differential equations (2-.5) and using the initial conditions at time t=0, $P_{Ou}(0)=1$ and all other initial condition probabilities are equal to zero, solving for *Liu[iu'] (s)*, we get

$$(s+\lambda+\alpha)\ p_{Ou}(s) - \gamma\ p_{ou'}(s) = 1\ ,$$
$$(s+\lambda+\alpha)\ p_{1U}(s) - \gamma\ p_{1U'}(s) = 0\ ,$$
$$(s+\lambda'+\gamma)\ P_{0U'}(s) - \alpha\ p_{0U}(s) = 0\ ,$$
$$(s+\lambda'+\gamma)\ p_{1U'}(s) - \alpha\ p_{1U}(s) = 0\ . \qquad (6\text{-}9)$$

and inverting, we get $p_i(t), i = OU, OU', 1U, 1U'$ .Then the system reliability is given by

$$R(t) = P_{OU}(t) + P_{1U}(t) + P_{OU'}(t) + P_{1U'}(t)$$
$$= \sum_{i=1}^{4} \frac{(s_i+\alpha+\gamma+\lambda')\left(s_i^2+\alpha\lambda'+(\gamma+\lambda')\lambda+s_i(\alpha+\gamma+\lambda'+\lambda)\right)}{\prod_{j=1, j\neq i}^{4}(s_i-s_j)} e^{s_i t} \qquad (10)$$

Where s_1, s_2, s_3, s_4 are the roots of the following equation?

$$\left(\alpha\gamma-(s+\gamma+\lambda')(s+\alpha+\lambda)\right)^2$$

Mean time to system failure

The steady-state reliability of the system is given by

$$R(s) = P_{OU}(s) + P_{1U}(s) + P_{OU'}(s) + P_{1U'}(s)$$

$$\underline{(S+\alpha+\gamma+\lambda')\ (S^2+\alpha\ \lambda'+(\gamma+\lambda')\ \lambda+S\ (\alpha+\gamma+\lambda'+\lambda\ ))} = \qquad (11)$$

The mean time to failure of the system is given by

$$MTTF = \lim_{s\to 0} R(s) = \frac{(\alpha+\gamma+\lambda')}{(\alpha\ \lambda'+(\gamma+\lambda')\ \lambda)} \qquad (12)$$

Variance transition of time to failure of the system

The variance transition of time to failure of the system is given by

$$\sigma^2 = -2\lim_{s\to 0} R'(s) - (MTTF)^2 = \frac{\alpha^2+(\gamma+\lambda')^2+2\alpha(\gamma+\lambda)}{(\alpha\eta+(\gamma+\lambda')\lambda)^2} \qquad (13)$$

Where $R'(s) = \frac{\delta R(s)}{\delta s}$

System availability

The system availability is the probability that system operates within the tolerances at a given instant of time and is obtained as follows: using the infinitesimal generator given in (1), we obtain the following differential equations

***Received**: 18-11-2024 - **Accepted**: 06-01-2025 - **Published**: 12-01-2025*

$$\frac{dp_{ou}}{dt} = -(\lambda+\alpha)\ p_{ou}(t) + \gamma\, p_{ou'}(t) + \beta\, p_{1D}(t)\ ,$$
$$\frac{dp_{1D}}{dt} = -(\beta+\mu+\alpha)\, p_{1D}(t) + \gamma\, p_{1D'}(t) + \lambda\, p_{OU}(t)\ ,$$
$$\frac{dp_{1U}}{dt} = -(\lambda+\alpha)\ p_{1U}(t) + \gamma\, p_{1U'}(t) + \mu\, p_{1D}(t) + \beta\ p_{2D}(t)\ ,$$
$$\frac{dp_{2D}}{dt} = -(\beta+\alpha)\ p_{2D}(t) + \gamma\, p_{2D'}(t) + \lambda\ p_{1U}(t)\ ,$$
$$\frac{dP_{0U'}}{dt} = -(\lambda'+\gamma)\ P_{0U'}(t) + \alpha\ p_{0U}(t) + \beta'\, P_{1D'}(t),$$
$$\frac{dp_{1D'}}{dt} = -(\mu'+\beta'+\gamma)\, p_{1D'}(t) + \alpha\ p_{1D}(t) + \lambda' p_{OU'}(t),$$
$$\frac{dp_{1U'}}{dt} = -(\lambda'+\gamma)\ p_{1U'}(t) + \alpha\ p_{1U}(t) + \mu'\ p_{1D'}(t) + \beta'\ p_{2D'}(t),$$
$$\frac{dp_{2D'}}{dt} = -(\beta'+\gamma)\ p_{2D'}(t) + \alpha\ p_{2D}(t) + \lambda'\ p_{1U'}(t). \tag{14-21}$$

and using the initial conditions at time t=0, $P_{OU}(0)=1$ and all other initial condition probabilities are equal to zero, solving for $L_i u[iu'](s)$ and $L_{iD}[iD'](s)$, we get

$$p_{OU}(s) = \frac{A(B+C)}{\prod_{r=1}^{8} s(s-s_r)},$$
$$p_{1U}(s) = \frac{(D+E)}{\prod_{r=1}^{8} s(s-s_r)},$$
$$p_{OU'}(s) = \frac{\alpha AF}{\prod_{r=1}^{8} s(s-s_r)},$$
$$p_{1U}'(s) = \frac{(G+H)}{\prod_{r=1}^{8} s(s-s_r)}. \tag{22-25}$$

Then we obtain

$$p_{OU}(t) = \frac{A_0(B_0+C_0)}{\prod_{r=1}^{8} s_r} + \sum_{i=1}^{8}\left(\frac{A_i(B_i+C_i)}{\prod_{r=1,r\neq i}^{8} s_i(s_i-s_r)}\right) e^{s_i t},$$
$$p_{1U}(t) = \frac{(D_0+E_0)}{\prod_{r=1}^{8} s_r} + \sum_{i=1}^{8}\left(\frac{(D_i+E_i)}{\prod_{r=1,r\neq i}^{8} s_i(s_i-s_r)}\right) e^{s_i t},$$
$$p_{OU'}(t) = \frac{\alpha A_0 F_0}{\prod_{r=1}^{8} s_r} + \sum_{i=1}^{8}\left(\frac{\alpha A_i F_i}{\prod_{r=1,r\neq i}^{8} s_i(s_i-s_r)}\right) e^{s_i t},$$

***Received**: 18-11-2024 - **Accepted**: 06-01-2025 - **Published**: 12-01-2025*

$$p_{1U'}(t) = \frac{(G_0+H_0)}{\prod_{r=1}^{8} s_r} + \sum_{i=1}^{8}\left(\frac{(G_i+H_i)}{\prod_{r=1,r\neq i}^{8} s_i(s_i-s_r)}\right)e^{s_i t}. \quad (26\text{-}29)$$

Since $OU, OU', 1U$ and $1U'$ correspond to system up-states, the system availability is given by

$$AV(t) = P_{OU}(t) + P_{1U}(t) + P_{OU'}(t) + P_{IU'}(t)$$

$$= \frac{1}{\prod_{r=1}^{8} s_r}\left[A_0(B_0+C_0+\alpha F_0)+D_0+E_0+G_0+H_0\right] + \sum_{i=1}^{8}\left(\frac{\left(A_i\left(B_i+C_i+\alpha F_i\right)+D_i+E_i+G_i+H_i\right)}{\prod_{r=1,r\neq i}^{8} s_i\left(s_i-s_r\right)}\right)e^{s_i t} \quad (30)$$

Steady-state availability

The steady-state availability of the system is given by

$$AV_0(\infty) = \lim_{s\to 0} sAV_0(s) = \frac{1}{\prod_{r=1}^{8}(-s_r)}\left[A_0(B_0+C_0+\alpha F_0)+D_0+E_0+G_0+H_0\right]$$

$$= \frac{\gamma+(\alpha+\gamma+\lambda')\beta+((\alpha+\gamma)(\alpha+\beta)+\alpha\lambda)\beta'}{(\alpha+\gamma)(\alpha(\lambda'+\beta')+(\beta+\lambda)(\gamma+\lambda'+\beta'))} \quad (31)$$

Where

$\prod_{r=1}^{8}(-s_r)$ are the roots of the equation (14-21), after taking Laplace transform,

$$A_i = s(s+\alpha+\gamma)(s^2+\alpha(\lambda'+\beta')+(\lambda+\beta)(\gamma+\lambda'+\beta')+s(\alpha+\gamma+\lambda+\beta+\lambda'+\beta'))$$

$$B_i = s(\gamma^2+(\beta+\mu)(\lambda'+\beta')+(\lambda'+\beta'+\mu)\mu'+\alpha(\gamma+\lambda'+\beta'+\mu')+\gamma(\lambda'+2\beta+2\mu+\beta'+\mu'))$$

$$C_i = \left(s^3+\gamma(\gamma+\lambda')(\beta+\mu)+\gamma(\alpha+\beta+\mu)\beta'+(\gamma+\lambda')(\alpha+\beta+\mu)\mu'+s^2(\alpha+2\gamma+\lambda'+\beta+\mu+\beta'+\mu')\right)$$

$$D_i = \lambda'\left(\mu\left((s+\gamma+\lambda')(s(s+\gamma+\alpha)+(s+\gamma)\beta)+(s+\gamma)(s+\alpha+\beta)\beta'\right)\right.$$

$$\left.(\alpha\gamma-\lambda\beta')+\alpha\gamma(s+\alpha+\beta+\mu)\left(s^2+\alpha\beta'+\beta(\gamma+\lambda'+\beta')+s(\alpha+\gamma+\beta+\beta')\right)\mu'\right)$$

$$E_i = (-s-\gamma-\lambda')\lambda\left(-\alpha\gamma\left(-\alpha\gamma+\lambda'\beta+(s+\alpha+\beta)(s+\gamma+\beta')\right)\mu'\right.$$

$$\left.-\mu\left((s+\gamma+\lambda')(s(s+\alpha+\gamma)+(s+\gamma)\beta)+(s+\gamma)(s+\alpha+\beta)\beta'\right)(s+\gamma+\beta'+\mu')\right)$$

$$F_i = \left(-\alpha\gamma+\lambda\beta'+(s+\alpha+\beta+\mu)(s+\gamma+\beta'+\mu')\right),$$

$$G_i = (s+\gamma+\beta')\left(\alpha(s+\alpha)(s+\alpha+\beta+\lambda)\left((s+\gamma+\lambda)\lambda+\lambda'(s+\alpha+\beta+\mu)\right)\mu'\right.$$

$$\left.-\alpha(s+\alpha+\beta)\mu\left(\lambda'(-\alpha\gamma+\lambda\beta')-(s+\gamma+\lambda')\lambda(s+\gamma+\beta'+\mu')\right)\right)$$

$$H_i = \alpha\left(-\lambda\mu\beta'(\lambda'(-\alpha\gamma+\lambda\beta')-(s+\gamma+\lambda')\lambda(s+\gamma+\beta'+\mu'))+\gamma\left((s+\alpha+\lambda)\left(-\alpha(s+\gamma+\lambda')\right.\right.\right.$$

$$\left.\left.\left.\lambda-\alpha\lambda'(s+\alpha+\beta+\mu)\right)\mu'+\alpha\mu\left(\lambda'(-\alpha\gamma+\lambda\beta')-(s+\gamma+\lambda')\lambda(s+\gamma+\beta'+\mu')\right)\right)\right)$$

since $A_i = A_0 \Rightarrow s_i = 0$, $A_i = A \Rightarrow s_i = s$.

Received: *18-11-2024* - ***Accepted***: *06-01-2025* - ***Published***: *12-01-2025*

Model II

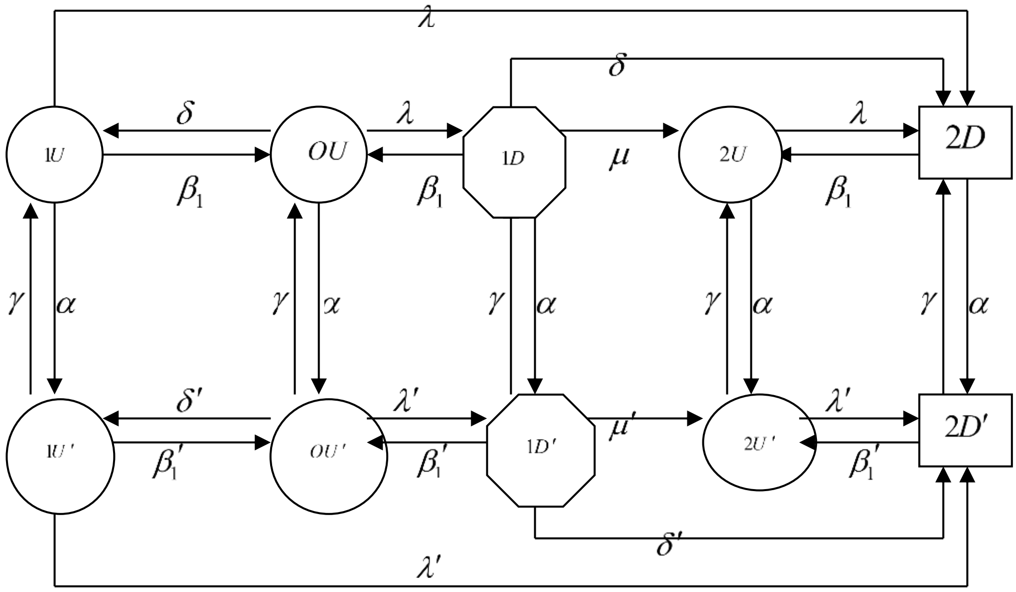

Figure 2. State space diagram for Model II

Using the transitions of the Markov process to the up states of the system. Let $P_{iU[iU']}(t), i = 0,1,2$ *and* $P_{iD[iD']}(t),\ i = 1,2$ be the probability that the system is in state $iU[iU'], i = 0,1,2$ *and* $iD[iD'], i = 1,2$ at time t. The infinitesimal generator of the Markove process is given below

$$Q_2 = \begin{bmatrix} -X_1 & \beta & \beta & 0 & 0 & \gamma & 0 & 0 & 0 & 0 \\ \delta & -X_2 & 0 & 0 & 0 & 0 & \gamma & 0 & 0 & 0 \\ \lambda & 0 & -X_3 & 0 & 0 & 0 & 0 & \gamma & 0 & 0 \\ 0 & 0 & \mu & -X_4 & \beta & 0 & 0 & 0 & \gamma & 0 \\ 0 & \lambda & \delta & \lambda & -X_5 & 0 & 0 & 0 & 0 & \gamma \\ \alpha & 0 & 0 & 0 & 0 & -Y_1 & \beta' & \beta' & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 & \delta' & -Y_2 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & \lambda' & 0 & -Y_3 & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 & 0 & 0 & \mu' & -Y_4 & \beta' \\ 0 & 0 & 0 & 0 & \alpha & 0 & \lambda' & \delta' & \lambda' & -Y_5 \end{bmatrix} \quad (32)$$

Where

$$X_1 = (\delta + \lambda + \alpha), X_2 = (\beta + \lambda + \alpha), X_3 = (\beta + \delta + \mu + \alpha), X_4 = (\alpha + \lambda) \text{ and } X_5 = (\beta + \alpha)$$
$$Y_1 = (\lambda' + \delta' + \gamma), Y_2 = (\beta' + \lambda' + \gamma), Y_3 = (\beta' + \mu' + \delta' + \gamma), Y_4 = (\lambda' + \gamma) \text{ and } Y_5 = (\beta' + \gamma)$$

We assume that initially both the units are operable and obtain the measures of system performance.

System reliability

The system reliability R(t) is the probability of failure-free operation of the system in (0, t]. To derive an expression for the reliability of the system, we restrict the transitions of the Markov process to the up states, viz. $iU[iU'],\quad i = o,1,2$ and. Using the infinitesimal generator given in (32), pertaining to these states and standard probabilistic arguments, we derive the following differential equations

$$\frac{dp_{OU}}{dt} = -(\delta + \lambda + \alpha)\ p_{OU}(t) + \beta\, p_{1U}(t) + \gamma\, p_{OU'}(t),$$

***Received**: 18-11-2024 - **Accepted**: 06-01-2025 - **Published**: 12-01-2025*

$$\frac{dp_{1U}}{dt} = -\left(\beta_1 + \lambda + \alpha\right) p_{1U}(t) + \delta p_{0U}(t) + \gamma p_{1U'}(t),$$
$$\frac{dp_{2U}}{dt} = -\left(\lambda + \alpha\right) p_{2U}(t) + \gamma p_{2U}'(t),$$
$$\frac{dp_{OU'}}{dt} = -\left(\delta' + \lambda' + \gamma\right) p_{OU'}(t) + \beta' p_{1U'}(t) + \alpha\, p_{OU}(t),$$
$$\frac{dp_{1U'}}{dt} = -\left(\beta' + \lambda' + \gamma\right) p_{1U'}(t) + \delta' p_{OU'}(t) + \alpha\, p_{1U}(t),$$
$$\frac{dp_{2U'}}{dt} = -\left(\lambda' + \gamma\right) p_{2U'}(t) + \alpha\, p_{2U}(t). \qquad (33\text{-}38)$$

Let $L_i(s)$ be the Laplace transform of $p_{iU[iU']}(t), i = 0,1,2$ .Taking Laplace transform on both the sides of the differential equations (33-38) and using the initial conditions at time t=0 , $P_{OU}(0) = 1$ and all other initial condition probabilities are equal to zero , solving for $L_{i}u[iu'](s)$; ,we get

$$\left(s + \delta + \lambda + \alpha\right) p_{OU}(s) - \beta p_{1U}(s) - \gamma p_{OU'}(s) = 1,$$
$$\left(s + \beta_1 + \lambda + \alpha\right) p_{1U}(s) - \delta p_{0U}(s) - \gamma p_{1U'}(s) = 0,$$
$$\left(s + \lambda + \alpha\right) p_{2U}(s) + \gamma p_{2U}'(s) = 0,$$
$$\left(s + \delta' + \lambda' + \gamma\right) p_{OU'}(s) - \beta' p_{1U'}(t) - \alpha\, p_{OU}(t) = 0,$$
$$\left(s + \beta' + \lambda' + \gamma\right) p_{1U'}(s) - \delta' p_{OU'}(s) - \alpha\, p_{1U}(s) = 0,$$
$$\left(s + \lambda' + \gamma\right) p_{2U'}(s) - \alpha\, p_{2U}(s) = 0. \qquad (39\text{-}44)$$

and inverting, we get $p_i(t), i = OU, OU', 1U, 1U', 2U, 2U'$ .Then the system reliability is given by

$$R(t) = \sum_{i=0}^{2} P_{iU}(t) + \sum_{i=0}^{2} P_{iU'}(t)$$
$$= \sum_{i=1}^{6} \frac{\left(s_i + \alpha + \gamma + \lambda'\right) Z_i}{\prod_{j=1, j\neq i}^{6} \left(s_i - s_j\right)} e^{s_i t} \qquad (45)$$

Where

$$Z_i = \left(s_i^2 + \alpha\lambda' + \left(\gamma + \lambda'\right)\lambda + s_i\left(\alpha + \gamma + \lambda + \lambda'\right)\right)\left(s_i^2 + \gamma\left(\delta + \beta + \lambda\right)\right.$$
$$\left. + \left(\alpha + \delta + \beta + \lambda\right)\left(\delta' + \lambda' + \beta'\right) + s_i\left(\alpha + \gamma + \delta + \delta' + \lambda + \lambda' + \beta + \beta'\right)\right)$$

$s_1, s_2, s_3, s_4, s_5, s_6$ are the roots of the following equation:

$$\left(-\alpha\gamma + \left(s + \gamma + \lambda'\right)\left(s + \alpha + \lambda\right)\right) Z$$

Mean time to system failure

The steady-state reliability of the system is given by

$$R(s) = \sum_{i=0}^{2} P_{iU}(s) + \sum_{i=0}^{2} P_{iU'}(s)$$
$$= \frac{(S + \alpha + \gamma + \lambda')}{(-\alpha\,\gamma + \left(S + \gamma + \lambda'\right)(S + \alpha + \lambda))} \qquad (46)$$

The mean time to failure of the system is given by

$$MTTF = \lim_{s\to 0} R(s) = \frac{(\alpha + \gamma + \lambda')}{(\alpha\,\lambda' + (\gamma + \lambda')\,\lambda)} \qquad (47)$$

Received: 18-11-2024 - **Accepted**: 06-01-2025 - **Published**: 12-01-2025

Variance transition of time to failure of the system

The variance transition of time to failure of the system is given by

$$\sigma^2 = -2\lim_{s\to 0} R'(s) - (MTTF)^2 = \frac{\alpha^2 + (\gamma+\lambda')^2 + 2\alpha(\gamma+\lambda)}{(\alpha\eta + (\gamma+\lambda')\lambda)^2} \quad (48)$$

Where $R'(s) = \dfrac{\delta R(s)}{\delta s}$

SYSTEM AVAILABILITY

The system availability is the probability that system operates within the tolerances at a given instant of time and is obtained as follows: using the infinitesimal generator given in (32), we obtain the following differential equations

$$\frac{dp_{OU}}{dt} = -(\delta+\lambda+\alpha)\, p_{OU}(t) + \beta\, p_{1U}(t) + \beta\, p_{2U}(t) + \gamma\, p_{OU'}(t),$$

$$\frac{dp_{1U}}{dt} = -(\beta+\lambda+\alpha)\, p_{1U}(t) + \delta p_{OU}(t) + \gamma p_{1U'}(t),$$

$$\frac{dp_{1D}}{dt} = -(s+\beta+\delta+\mu+\alpha)\, p_{1D}(t) + \lambda\, p_{OU}(t) + \gamma p_{1D'}(t),$$

$$\frac{dp_{2U}}{dt} = -(\lambda+\alpha)\, p_{2U}(t) + \mu\, p_{1D}(t) + \beta\, p_{2D}(t) + \gamma\, p_{2U'}(t),$$

$$\frac{dp_{2D}}{dt} = -(\beta_1+\alpha)\, p_{2D}(t) + \lambda\, p_{1U}(t) + \delta\, p_{1D}(t) + \lambda\, p_{2U}(t) + \gamma\, p_{2D'}(t),$$

$$\frac{dp_{OU'}}{dt} = -(s+\delta'+\lambda'+\gamma)\, p_{OU'}(t) + \beta'\, p_{1U'}(t) + \beta'\, p_{2U'}(t) + \alpha\, p_{OU}(t),$$

$$\frac{dp_{1U'}}{dt} = -(\beta'+\lambda'+\gamma)\, p_{1U'}(t) + \delta'\, p_{0U'}(t) + \alpha\, p_{1U'}(t),$$

$$\frac{dp_{1D'}}{dt} = -(\mu'+\delta'+\beta_1'+\gamma)\, p_{1D'}(t) + \lambda' p_{0U'}(t) + \alpha\, p_{1D}(t),$$

$$\frac{dp_{2U'}}{dt} = -(\lambda'+\gamma)\, p_{2U'}(t) + \mu'\, p_{1D'}(t) + \beta'\, p_{2D'}(t) + \alpha\, p_{2U}(t)$$

$$\frac{dp_{2D'}}{dt} = -(\beta_1'+\gamma)\, p_{2D'}(t) + \lambda'\, p_{1U'}(t) + \delta'\, p_{1D'}(t) + \lambda' p_{2U'}(t) + \alpha\, p_{2D}(t). \quad (49\text{-}55)$$

and using the initial conditions at time t=0 , $P_{ou}(0)=1$ and all other initial condition probabilities are equal to zero , solving for $L_i u[iu'](s)$ and $L_{iD}[_{iD}'](s)$; ,we get

$$p_{OU}(s) = \frac{AQ}{\prod_{r=1}^{10} s(s-s_r)},$$

$$p_{1U}(s) = \frac{AQ'}{\prod_{r=1}^{10} s(s-s_r)},$$

$$p_{2U}(s) = \frac{Q''}{\prod_{r=1}^{10} s(s-s_r)},$$

$$p_{OU'}(s) = \frac{\alpha AQ'''}{\prod_{r=1}^{10} s(s-s_r)},$$

***Received**: 18-11-2024 - **Accepted**: 06-01-2025 - **Published**: 12-01-2025*

$$p_{1U'}(s)=\frac{\alpha A Q''''}{\prod_{r=1}^{10} s\left(s-s_r\right)},$$

$$p_{2U'}(s)=\frac{A Q'''''}{\prod_{r=1}^{10} s\left(s-s_r\right)}. \qquad (56\text{-}61)$$

Then we obtain

$$p_{OU}(t)=\frac{A_0 Q_0}{\prod_{r=1}^{10} s_r}+\sum_{i=1}^{10}\left(\frac{A_i Q_i}{\prod_{r=1,r\neq i}^{10} s_i\left(s_i-s_r\right)}\right)e^{s_i t},$$

$$p_{1U}(t)=\frac{A_0 Q_0'}{\prod_{r=1}^{10} s_r}+\sum_{i=1}^{10}\left(\frac{A_i Q_i'}{\prod_{r=1,r\neq i}^{10} s_i\left(s_i-s_r\right)}\right)e^{s_i t},$$

$$p_{2U}(t)=\frac{Q_0''}{\prod_{r=1}^{10} s_r}+\sum_{i=1}^{10}\left(\frac{Q_i''}{\prod_{r=1,r\neq i}^{10} s_i\left(s_i-s_r\right)}\right)e^{s_i t},$$

$$p_{OU'}(t)=\frac{\alpha A_0 Q_0'''}{\prod_{r=1}^{10} s_r}+\sum_{i=1}^{10}\left(\frac{\alpha A_i Q_i'''}{\prod_{r=1,r\neq i}^{10} s_i\left(s_i-s_r\right)}\right)e^{s_i t},$$

$$p_{1U'}(t)=\frac{\alpha A_0 Q_0''''}{\prod_{r=1}^{10} s_r}+\sum_{i=1}^{10}\left(\frac{\alpha A_i Q_i''''}{\prod_{r=1,r\neq i}^{10} s_i\left(s_i-s_r\right)}\right)e^{s_i t},$$

$$p_{2U'}(t)=\frac{A_0 Q_0'''''}{\prod_{r=1}^{10} s_r}+\sum_{i=1}^{10}\left(\frac{A_i Q_i'''''}{\prod_{r=1,r\neq i}^{10} s_i\left(s_i-s_r\right)}\right)e^{s_i t}. \qquad (62\text{-}67)$$

Since $OU, OU', 1U, 1U', 2U, 2U'$ correspond to system up-states, the system availability is given by

$$AV(t)=P_{OU}(t)+P_{1U}(t)+P_{2U}(t)+P_{OU'}(t)+P_{IU'}(t)+P_{IU'}(t)$$

$$=\frac{Q_0''+A_0\left(Q_0+Q_0'+\alpha Q_0'''+\alpha Q_0''''+Q_0'''''\right)}{\prod_{r=1}^{10} s_r}+\sum_{i=1}^{10}\left(\frac{Q_i''+A_i\left(Q_i+Q_i'+\alpha Q_i'''+\alpha Q_i''''+Q_i'''''\right)}{\prod_{r=1,r\neq i}^{10} s_i\left(s_i-s_r\right)}\right)e^{s_i t} \qquad (68)$$

***Received**: 18-11-2024 - **Accepted**: 06-01-2025 - **Published**: 12-01-2025*

https://doi.org/10.54361/ajmas.258112

Steady-state availability

The steady-state availability of the system is given by

$$AV_0(\infty)=\lim_{s\to 0} sAV_0(s)=\frac{Q_0''+A_0\left(Q_0+Q_0'+\alpha Q_0'''{}'+\alpha Q_0''''{}'+Q_0{}''''{}''\right)}{\prod_{r=1}^{10}(-s_r)}$$

$$=\frac{\gamma+(\alpha+\gamma+\lambda')\beta+((\alpha+\gamma)(\alpha+\beta)+\alpha\lambda)\beta'}{(\alpha+\gamma)\left(\alpha(\lambda'+\beta')+(\beta+\lambda)(\gamma+\lambda'+\beta')\right)} \quad (69)$$

Where

$\prod_{r=1}^{10}(-s_r)$ are the roots of the equation (49-55) after taking Laplace transform

$$Q=\left(-\alpha\gamma K1+(s+\alpha+\delta+\beta+\mu)\left(-\lambda'\beta'\left(-\alpha\gamma+(s+\alpha+\beta+\lambda)\right.\right.\right.$$
$$\left.\left.\left.(s+\gamma+\lambda'+\beta')\right)+K1(s+\gamma+\delta'+\beta'+\mu')\right)\right)$$

$$Q'=A\left(K2+\delta'\left(K3+\alpha\left(\gamma^2(\delta+\beta+\mu)-\delta\beta'(\delta'+\beta'+\mu')+\gamma\left(\delta'(\beta+\mu)\right.\right.\right.\right.$$
$$\left.\left.\left.\left.+(\beta+\lambda+\mu)\beta'+(\beta+\mu)\mu'+\delta(\delta'+\beta'+\mu')\right)\right)\right)\right)$$

$$Q''=-\gamma\left(-K4(\alpha\gamma-\lambda'\beta)\mu'-(s+\gamma+\lambda')(-\alpha K6+\beta K7)\right)$$
$$-\beta'\left(\gamma\left(-(s+\alpha+\beta)K7+\alpha K5\right)-\lambda'\left(-(s+\alpha+\beta)K6+\beta K5\right)\right)-(s+\gamma+\beta')\left(\gamma\mu'(s+\alpha+\beta)K4\right.$$
$$\left.+(s+\gamma+\lambda')\left(-(s+\alpha+\beta)K6+\beta K5\right)\right)$$

$$Q'''=\left(-\alpha^2\gamma(\lambda'+\beta')+\lambda(\beta+\lambda)\beta'(\gamma+\lambda'+\beta')-K8-(s+\alpha+\delta+\beta+\mu)\right.$$
$$\left.\left(\alpha\gamma-\delta\beta'-(s+\alpha+\beta+\lambda)(s+\gamma+\lambda'+\beta')\right)(s+\gamma+\delta'+\beta'+\mu')\right)$$

$$Q''''{}'=\left(-\delta'(s+\alpha+\beta+\lambda)\left(\alpha\gamma-\lambda\beta'-(s+\alpha+\delta+\beta+\mu)(s+\gamma+\delta'+\beta'+\mu')\right)\right.$$
$$\left.+\delta\left(-\lambda'(s+\alpha+\delta+\beta+\mu)\beta'+(s+\gamma+\delta'+\lambda')\left(-\alpha\gamma+(s+\alpha+\delta+\beta+\mu)(s+\gamma+\delta'+\beta'+\mu')\right)\right)\right)$$

$$Q''''{}''=\alpha\gamma\left((s+\alpha+\lambda)K4\mu'-\alpha k7+\beta k6\right)+\beta'\left(-\lambda(-\alpha k7+\beta k6)-(s+\alpha+\lambda)\right.$$
$$\left.\left((-s-\alpha-\beta)K6+\alpha k5\right)\right)+(s+\gamma+\beta')\left(\alpha(s+\alpha)(s+\alpha+\beta+\lambda)K9\mu'-\alpha\left((-s-\alpha-\beta)K7+\beta K5\right)\right)$$

$$K1=\left((S+\gamma+\delta'+\lambda')\left(s^2+\alpha\lambda'+(\gamma+\lambda')(\beta+\lambda)+s(\alpha+\gamma+\lambda'+\beta+\lambda)\right)\right.$$
$$\left.+(s+\gamma+\lambda')(s+\alpha+\beta+\lambda)\beta'\right)$$

$$K2=\delta(s+\gamma+\lambda'+\beta')\left(-\lambda'(s+\alpha+\delta+\beta+\mu)\beta'+(s+\gamma+\delta'+\lambda')\right.$$
$$\left.\left(-\alpha\gamma+(s+\alpha+\delta+\beta+\mu)(s+\gamma+\delta'+\beta'+\mu')\right)\right)$$

$$K3=s^2(\alpha\gamma-\delta\beta')+\alpha^2\gamma(\delta'+\beta'+\mu')-\delta(\delta+\beta+\mu)\beta'(\gamma+\delta'+\beta'+\mu')+$$
$$s(\alpha\gamma-\delta\beta')(\alpha+\gamma+\delta+\delta'+\beta+\beta'+\mu+\mu')$$

$$k4=-\alpha\delta'(s+\alpha+\delta+\beta+\mu)\left(-\alpha\gamma+\delta\beta'+(s+\alpha+\beta+\lambda)(s+\gamma+\lambda'+\beta')\right)$$
$$-\alpha\lambda\left(-\delta'(s+\alpha+\beta+\lambda)\beta'+(s+\gamma+\delta'+\lambda')\left(-\alpha\gamma+(s+\alpha+\beta+\lambda)(s+\gamma+\lambda'+\beta')\right)\right)$$

***Received**: 18-11-2024 - **Accepted**: 06-01-2025 - **Published**: 12-01-2025*

$$k5=-\delta\left(-\lambda'\left(\left(s+\alpha+\beta+\lambda\right)\left(s+\lambda+\lambda'+\beta'\right)\left(-\alpha\gamma+\lambda\beta'\right)+\right.\right.$$
$$\alpha\gamma\left(\alpha\gamma-\left(\delta+\lambda\right)\beta'\right)\Big)+\lambda\left(-\delta'\left(s+\alpha+\beta+\lambda\right)\beta'+\left(s+\gamma+\delta'+\lambda'\right)\right.$$
$$\left(-\alpha\gamma+\left(s+\alpha+\beta+\lambda\right)\left(s+\gamma+\lambda'+\beta'\right)\right)\Big)\left(s+\gamma+\delta'+\beta'+\mu'\right)\Big)+\lambda\Big(\delta'\left(\alpha\gamma\left(\alpha\lambda-\left(\delta+\lambda\right)\beta'\right)\right.$$
$$+\left(s+\alpha+\delta+\beta+\mu\right)\left(-\alpha\gamma+\delta\beta'\right)\left(s+\gamma+\delta'+\beta'+\mu'\right)\Big)-\delta\left(s+\gamma+\lambda'+\beta'\right)$$
$$\left(-\lambda'\left(s+\alpha+\delta+\beta+\mu\right)\beta'+\left(s+\gamma+\delta'+\lambda'\right)\left(-\alpha\gamma+\left(s+\alpha+\delta+\beta+\mu\right)\left(s+\gamma+\delta'+\beta'+\mu'\right)\right)\right)\Big)$$
$$K6=-K4\delta'+\lambda'\Big(\alpha\delta'\left(s+\alpha+\beta+\lambda\right)\left(-\alpha\gamma+\lambda\beta'+\left(s+\alpha+\delta+\beta+\mu\right)\left(s+\gamma+\delta'+\beta'+\mu'\right)\right)$$
$$+\alpha\delta\left(-\lambda'\left(s+\alpha+\delta+\beta+\mu\right)\beta'+\left(s+\gamma+\delta'+\lambda'\right)\left(-\alpha\gamma+\left(s+\alpha+\delta+\beta+\mu\right)\left(s+\gamma+\delta'+\beta'+\mu'\right)\right)\right)\Big)$$
$$K7=\mu\Big(-\lambda'\left(\left(s+\alpha+\beta+\lambda\right)\left(s+\gamma+\lambda'+\beta'\right)\left(-\alpha\gamma+\lambda\beta'\right)+\alpha\gamma\left(\alpha\gamma-\left(\delta+\lambda\right)\beta'\right)\right)$$
$$+\lambda\left(-\delta'\left(s+\alpha+\beta+\lambda\right)\beta'+\left(\left(s+\gamma+\delta'+\lambda'\right)\left(-\alpha\gamma+\left(s+\alpha+\beta+\lambda\right)\left(s+\gamma+\lambda'+\beta'\right)\right)\right)\right)$$
$$\left(s+\gamma+\delta'+\beta'+\mu'\right)\Big)$$
$$K8=s^2\left(\alpha\gamma-\lambda\beta'\right)-s\left(\alpha+\gamma+\lambda'+\beta+\lambda+\beta'\right)\left(\alpha\gamma-\lambda\beta'\right)-\alpha\Big(\gamma^2\left(\beta+\lambda\right)$$
$$+\gamma\lambda'\left(\beta+\lambda\right)+\gamma\left(\delta+\beta+\lambda\right)\beta'-\lambda\beta'\left(\lambda'+\beta'\right)\Big)$$
$$K9=\left(-\lambda'\left(s+\alpha+\delta+\beta+\mu\right)\left(\alpha\gamma-\delta\beta'-\left(s+\alpha+\beta+\lambda\right)\left(s+\gamma+\lambda'+\beta'\right)\right)+\lambda\Big(-\delta'\left(s+\alpha+\beta+\lambda\right)\beta'\right.$$
$$+\left(s+\gamma+\delta'+\lambda'\right)\left(-\alpha\gamma+\left(s+\alpha+\beta+\lambda\right)\left(s+\gamma+\lambda'+\beta'\right)\right)\Big)\Big)$$

since $Q_j^r=Q_0^r\Rightarrow s_i=0$, $Q_j^r=Q^r\Rightarrow s_i=s$.

Conclusion

We conclude that the steady-state availability first system and second system. The mean time to failure of the system for two systems and the variance. Transition of time to failure of the two systems we obtained on the second model not differs from the first model for all parameters.

Conflict of interest. Nil

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***Received**: 18-11-2024 - **Accepted**: 06-01-2025 - **Published**: 12-01-2025*

المستخلص

في هذه الورقة تقدم تحليل الموثوقية لنموذجين رياضـــيين يمثلان أنظمة الطاقة الكهربائية العاملة في طقس خارجي متقلب (أي الطقس العادي والعاصــف) ومقارنة بين نموذجين. يتعامل النموذج الأول مع تحليل الموثوقية لوحدة احتياطية باردة ذات خادم واحد ووحدتين؛ ويتعامل النموذج الثاني مع تحليل الموثوقية لوحدة احتياطية دافئة ذات خادم واحد ووحدتين. لنظامين بنمطين مختلفين [طبيعي، فشـــل كلي]. يحدث فشـــل النظام عندما تفشـل كلتا الوحدتين تمامًا. معدل الفشـل ومعدل إصلاح الوحدة الفاشـلة ثابتان. تم اشـتقاق تحويلات لابلاس لاحتمالات الحالة المختلفة ومن ثم تم الحصول على الموثوقية من خلال عملية العكس. علاوة على ذلك، تم اشـتقاق معلمة مهمة للموثوقية، أي متوسـط الوقت حتى الفشـل، وانتقال التباين لوقت فشل النظام، وتوافر النظام وتوافر الحالة المستقرة. يتم توزيـع أوقات فشل الوحدات العاملة/الاحتياطية ووقت إصلاح الوحدات الفاشلة بشكل أسي. تمت مقارنة بعض النتائج المهمة بين نظامين.

Received*: 18-11-2024 -* ***Accepted****: 06-01-2025 -* ***Published****: 12-01-2025*