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Original article

Application of Graph Theory to Analyze Gaps in Water and Utility Networks to Enhance the Resilience of Urban Infrastructure

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Abstract
This work investigates the potential application of graph theory to understand the vulnerability of urban water and utility networks (graph theory provides opportunities to enable resilience in urban water and utility networks) and models urban water and utilities as a structurally functional model of the main water and utility components as graphs, with the critical components (water treatment plant, pumping station and/or distribution terminal) being the nodes and the pipes, power lines and/or other conduits connecting one to another being the edges. Graph theory can be used to diagnose systematically and more rigorously the most structurally vulnerable and operationally inefficient points in the last-mile logistics of complex infrastructures. Some of the simplest graph-theoretical applications, including connectivity and centrality, clustering coefficients and network analysis tools, are used to identify critical disconnections, bottlenecks and single points of failure that can impact the performance of a water or utility network under stressors (such as natural disasters, cyberattacks and equipment failure). Data will also measure the level of connectivity of the networks, as well as how much less a utility truly is once a node/edge has been removed, and how to use graph theory to find these nodes/edges. The paper also highlighted practical steps to optimization that service providers can work on to become a resilient and continue providing services in the event of a failure, such as improving network redundancy, re-routing critical paths and prioritizing on upgrading. This work has led to a novel approach to modelling flexible and resilient infrastructural fabric that can withstand and be resilient to disruption in measurable and scalable ways using the incorporation of graph theory and urban planning. The recommendations provide valuable guidance for policymakers, engineers, city planners and others as they seek to advance future-proofing utility networks and services in the face of environmental and human impacts. The study provides a mathematically precise quantification of vulnerability and resilience by representing urban water and utility systems as weighted graphs in the mathematical literature and applying matrix representation, spectral graph theory and flow-based optimization. We are able to give explicit mathematical formulae for these connectivity and centrality metrics, as well as for those of robustness and resilience, which make the assessment of such infrastructure systems reproducible and give principles for improving them.
Keywords. Network Resilience, Urban Infrastructure, Resilience, Water Utility, Network Analysis.

Introduction

Urban infrastructure networks are considered to be the backbone of the modern town. It provides fundamental facilities that enable for the daily business and daily life of people and agencies. One of the critical offerings is water distribution and application networks, which contribute to economic fitness, public safety, and public health [1]. Urban spaces are becoming more complex as weather trends, herbal failures, cyberattacks, and aging infrastructure continue to create greater demands on the infrastructure networks to adjust. Emerging principles [2] require resilience thinking to develop, sustain and operate infrastructure networks. Graph is a branch of mathematics which can be used to evaluate the relation of mixed up objects. It can present methods to measure the structural and functional properties of infrastructure networks [3].

Graph-based infrastructure analysis revolutionizes the interpretation of the design and optimization of any infrastructure network by engineers and planners. The traditional way would be to monitor the premises physically, where the issues are happening, and to make an active maintenance, which could not be aware of the structural susceptibility of the network [4]. In contrast, graph-theoretic methods provide a larger picture, defining the topology of the network and the most important elements. All these aspects can collapse and their collapse can have a domino effect on the system [5]. As cities advance in complexity, they become interdependent, interconnected, and detached due to brutal emergent threats; more mathematical rigor is used in the management and planning of resilience in urban areas [6].

The water distribution system is one of the major problems of the modern urban infrastructure. These are made up of thousands of assets, which are interconnected with each other and some examples of these are storage tanks, treatment facilities, pumping stations, valves and distribution piping [7]. These discrete, individual networks must run around the clock to be capable of delivering clean water to millions of consumers, as well as meet multiple performance requirements like pressure, quality and redundancy for disruptions [8]. Water distribution systems are susceptible to a wide range of hazards including equipment failures and pipe blockages, as well as pollution and natural disasters. This drawback emphasizes the significance of analysis tools leading to forecasting and preventing a system breakdown [9].

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Literature Review

In the last 20 years, the academic literature has been interested in graph theory and infrastructure network analysis, as we try to use the power of network science for urban problems [10]. The founders of graph theory have also demonstrated that two simple graph measures (degree distribution, path length) can be used to approximate key features of infrastructure networks and the resilience and vulnerability of infrastructure networks [11]. The previous study was included to guide the first step in the development of a theoretical framework on how users of graph theory could approach real infrastructure problems. It has also been shown that several utility networks have small-world or scale-free network structures, an important finding when one keeps in mind that we are depending on them to provide us with a service that is reliable and redundant [12].

Table 1. Change Over Time: Incorporating Graph Theory into Infrastructure Investigations.

Time Period	Key Developments	Key Areas of Focus	Key Contributions
1990-2000	Basic network modeling	Topology description	Scale-free networks introduced [13]
2000-2010	Vulnerability assessment	Identification of critical nodes	Centrality metrics developed [14]
2010-2020	Resilience enhancement	Cascading failures	Geographic information systems (GIS) integration [15]
2020-Present	AI-assisted review/analysis	Predictive maintenance	Machine learning integration [16]

Based on what we have covered, researchers have applied both the structure of network models and the model of load and capacity to each of these frameworks to support operational issues and problems in regard to distribution networks (water) and utility networks. It has been studied how different network topologies impact resilience, and it was found that increasing the degree of connectivity or redundancy of a water distribution network or an infrastructure network increased resilience to random failure and disruption (the definition of resilience). They still represent serious exposure to targeted exploration of network vulnerable or critical nodes, in line with Anderson (2016) [17]. This fact indicates that network analysis is not only needed in terms of statistical properties, but also in terms of functional nature and specific threats to the functioning of networks [18]. Another important development is the development of resilience metrics that are specific to infrastructure networks. Researchers have developed various resilience measures. These measures are of much more importance to engineers, including top-level connectivity measures. The variables of interest identified in the empirical study of water distribution systems to consider when thinking of resilience are the following: Flow capacity, Level of Service in time, and recovery of Level of service within a given time [19]. It thus helps enhance an assessment of the system performance, which is beneficial for capex spending decision-making and/or for what to take into consideration in maintenance/rehabilitation priorities [20].

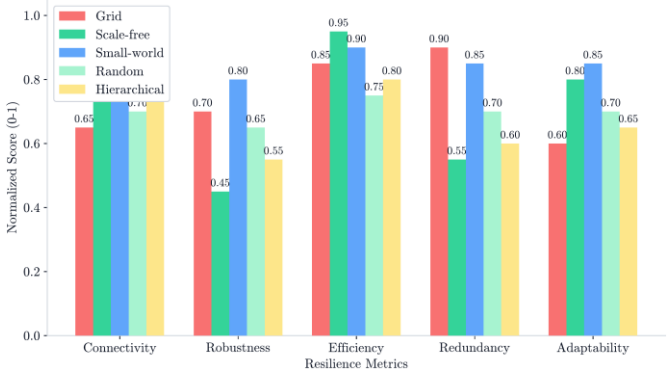


Figure 1. Comparison of Network Resilience Metrics Across Different Network Types.

More recently, more emphasis has been placed on interdependencies between infrastructure systems, as we realize that the contemporary city is based on complex networks of networks – where disruptions can transfer through various infrastructure systems [21]. For instance, water distribution systems rely on electrical energy to drive water through its pumping stations, while some energy production facilities use water for self-cooling. Both systems are interdependent, and any disruption in one system could decrease the resiliency of any dependent system [22]. Graph-theoretic approaches have been useful to analyze interdependencies between infrastructure systems and to identify critical coupling points for directed interventions to enhance system resilience [23]. Graph-theoretic and algebraic methods have been at the core of the understanding of the structural properties of networks for a long time. Algebraic graph representations such as adjacency matrices and characteristic polynomials offer profound insights into connectivity, eigenvalue spectra, and topological behavior. The relationship between structure and spectral properties of a graph has been studied with the

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characteristic polynomial of classical families of graphs like paths and cycles, where the vertex and edge configurations have been related to the vertex spectra and the global behavior of the graph [24].
In recent years, vertex-based incompatibility notions were introduced to study topological views of directed graphs, namely, incompatibility substructures of digraphs that are constructed from the vertices that are not adjacent or are structurally constrained [25]. The results of these studies indicate that vertex interactions need to be taken into account beyond mere adjacency, and that novel mathematical techniques like spectral analysis, centrality measures, and topological constraints are required in the study of real-world complex networks. Based on these theoretical foundations, the current research extends the concepts of algebraic and topological graph analysis to the evaluation and optimization of urban water and utility networks' resilience.

Theoretical Framework

We model the infrastructure as a weighted graph

$$G = (V, E, W), \tag{1}$$

where $V = \{v_1, \dots, v_n\}$ denotes infrastructure components, $E \subseteq V \times V$ denotes physical links, and $W: E \to \mathbb{R}_+$ assigns engineering weights such as capacity, cost, or risk.

The weighted adjacency matrix $A = [a_{ij}]$ is defined by

$$a_{ij} = \begin{cases} w_{ij}, & (v_i, v_j) \in E, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The degree matrix is $D = \mathrm{diag}(d_1, \dots, d_n)$ with $d_i = \sum_j a_{ij}$.

$$C_D(v_i) = \sum_{j=1}^{n} a_{ij}\,, \quad C_B(v_i) = \sum_{s \neq i \neq t} \frac{\sigma_{st}(i)}{\sigma_{st}}, \tag{3}$$

$$C_C(v_i) = \frac{n-1}{\sum_{j \neq i} d\left(v_i, v_j\right)}, \quad A\boldsymbol{x} = \lambda_{\max}\boldsymbol{x}. \tag{4}$$

The Laplacian matrix is

$$L = D - A. \tag{5}$$

Let $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ be the eigenvalues of L. The algebraic connectivity λ_2 measures robustness.

$$\max \sum_{(s,j) \in E} f_{sj} \tag{6}$$

$$\text{s.t. } 0 \leq f_{ij} \leq c_{ij}, \tag{7}$$

$$\sum_j f_{ij} - \sum_j f_{ji} = 0, \ \forall i \neq s, t. \tag{8}$$

$$R = \frac{1}{T}\int_0^T \frac{Q(t)}{Q_0}\, dt. \tag{9}$$

The continuous-time definition captures the extent and duration of service disruption, and offers a common frame of reference for comparing the resilience of various disruption scenarios and recovery actions. This research offers a theoretical underpinning in the sense that it follows some key principles from graph theory that can be adjusted according to the goals of an infrastructure network analysis. The graph G=(V, E) is a network topology representation made up of vertices V corresponding to network elements, and edges E corresponding to the links between the network elements [24]. In water distribution networks, the vertices are the physical infrastructure components: treatment plants, reservoirs, pumping stations, and demand nodes, while the edges are the pipes, channels, or other conduits that transport the water between the components.
The adjacency matrix, A, is a mathematical representation of the connectivity of a network. In this representation, the a_ij are 1 when there is an edge between vertices i and j and 0 otherwise. In certain practical applications, the unweighted representation of the A matrix in the form of a graph won't accurately represent the connections in the networks, especially in terms of capacity or importance when the connections are important because of different weightings or resources. Hence, for the weighted graph, the adjacency matrix can be used to represent adjacency with non-binary values, so the elements hold the edge weight information.
A key graph parameter that is relevant for assessing the connectivity of a particular vertex (V) is the degree of the vertex, denoted (d_v), which is where a vertex has the number of edges incident to it. Degree is a measure of local connectivity and helps to gauge vulnerability and resilience associated with network nodes. Understanding degree is important. Centrality measures are an efficient analysis method to identify critical elements in public infrastructure networks. Degree centrality is the number of connections on a node, or how connected a node is to other nodes. Betweenness centrality is the frequency of a node being on the shortest paths between nodes, which articulates the value of certain elements for flow and communication in the network. Closeness centrality measures the distance to or accessibility of a node in a network, which is the average distance of a node to all other nodes. It is an extension of degree centrality in the sense that it considers not only the number of connections a node has but also takes into account the connections of the connections of the node. You could think of it as a more sophisticated measure of influence, because it takes into account the structure of connections to other connections in the network.

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Table 2: Graph-Theoretic Metrics for Infrastructure Network Analysis.

Metric Category	Specific Measures	Infrastructure Relevance	Computational Complexity
Connectivity	Node degree, Edge connectivity	Redundancy assessment	O(V + E)
Centrality	Betweenness, Closeness, Eigenvector	Critical component identification	$O(V^3)$
Clustering	Local clustering coefficient	Community structure analysis	$O(V^3)$
Path Analysis	Shortest path, Network diameter	Flow optimization	$O(V^2 \log V)$
Robustness	Algebraic connectivity, Percolation threshold	Failure tolerance evaluation	$O(V^3)$

Theoretical Framework

Mathematical Preliminaries

An urban infrastructure system is modeled as a graph $G = (V, E)$, where $V = \{v_1, v_2, \dots, v_n\}$ denotes the finite set of vertices representing physical components such as treatment plants, reservoirs, pumping stations, and demand nodes, and $E \subseteq V \times V$ denotes the set of edges representing physical or functional connections between these components. In directed networks, an edge $(v_i, v_j) \in E$ represents a directed flow or dependency from v_i to v_j, while undirected edges represent bidirectional connections.

The structure of the graph is encoded by the adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$, defined as

$$a_{ij} = \begin{cases} w_{ij}, & \text{if } (v_i, v_j) \in E, \\ 0, & \text{otherwise,} \end{cases} \quad (10)$$

where $w_{ij} > 0$ denotes the weight of edge (v_i, v_j), representing attributes such as pipe capacity, flow potential, or operational importance. For unweighted graphs, $w_{ij} = 1$ whenever an edge exists.

The degree of a vertex v_i is defined as

$$d_i = \sum_{j=1}^{n} a_{ij}, \quad (11)$$

and the corresponding degree matrix is given by

$$D = \text{diag}(d_1, d_2, \dots, d_n). \quad (12)$$

Based on the adjacency and degree matrices, the (combinatorial) graph Laplacian is defined as

$$L = D - A. \quad (13)$$

The Laplacian matrix plays a fundamental role in the analysis of network connectivity, robustness, diffusion processes, and spectral properties, and serves as a key tool for assessing the resilience of infrastructure networks.

Network flow directly extends the concept of graph theory to how an infrastructure system operates. Maximum flow analysis can be described as maximizing the quantity of a resource that can be transported from a source node to a sink node. In minimum cost flow problems, we need to maximize the flow of a resource not only based on capacity limit conditions, but also based on the cost of operation. Flow-based analysis may also provide the added benefit of providing the user with visibility into system capacity conditions, capacity bottlenecks, and the optimal routes of preference required to plan and manage an infrastructure system.

Another route for a priori analysis is available from spectral graph theory via a graph's eigenvalues and eigenvectors. For example, the second smallest eigenvalue of the Laplacian matrix, referred to commonly as the algebraic connectivity or Fiedler value, can tell how "connected" a graph is, as well as how "resistant" it will be to becoming disconnected. The spectrum of the adjacency matrix provide insight into certain structural properties of the network, such as community identification, identification of bottlenecks and network synchronization or cascading failures.

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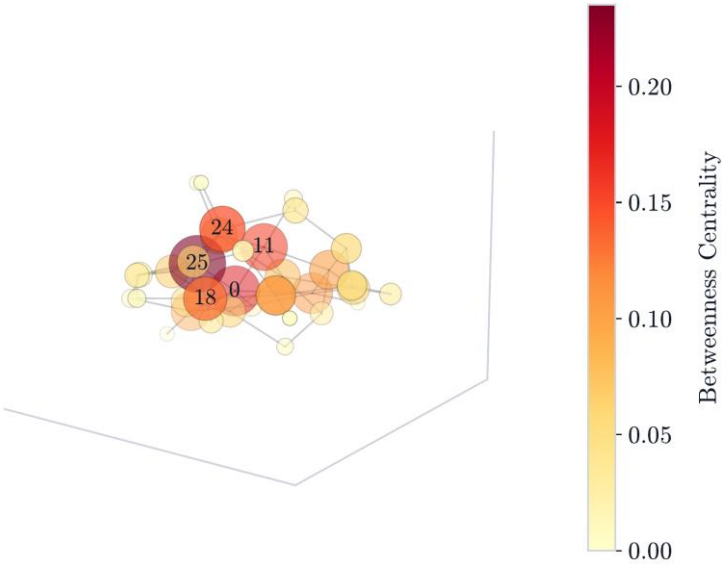


Figure 2. Multi-dimensional Centrality Analysis.

Methodology

The research approach employs theory-informed graphical analysis combined with current real-world infrastructure network data to generate comprehensive vulnerability assessments or resilience strengthening plans. The methodology comprises five main phases: (i) data collection and network modeling; (ii) structural analysis (graph metrics); (iii) vulnerability testing (failure or functional modelling); (iv) resilience enhancement and optimization; (v) case study validation of the approaches and assessments. The first model-driven step in the modeling of a network (or a coherent grouping of networks) is the transduction of practically accessible physical infrastructure data into graph-based forms to allow the execution of mathematical computations. It is also important to note here that any extraction to a graph-based representation of data will incur some degree of abstraction, so network-based models of infrastructure will need to be abstracted at least enough to be useful. They also want to avoid too much abstraction that prevents developing a computer scheme so complex that it defies proper computational functionality. The graph-based model should abstract wisely and avoid abstraction that loses critical features. Arc-shaped data will be geo-referenced to infrastructure components, and mapped spatial relationships between infrastructure components will reflect the spatial context to construct the network. From the perspective of graph-based methods, hydraulic models can provide modeling parameters (pipe capacity, flow directions, pressure requirements, etc.) which are important in informing the edge weights and the node information.

The proposed procedure integrates topological (centrality), spectral (algebraic connectivity), and operational (flow feasibility) criteria into a unified workflow. The ranking step produces a reproducible set of high-impact components, while the greedy augmentation stage provides a practical budget-feasible method for increasing the resilience functional. Data preprocessing is the process of correcting common abnormalities in infrastructure datasets. These abnormalities can be a result of missing data, formats that were changed, and the temporally distributed nature of the data from time to time. For example, when doing this task, interpolation may be used to fill in missing pipe diameter or material using values based on which neighbours had the related data or the relevant times of construction. Also, there are network simplification algorithms, which will decrease the complexity of the dataset by aggregating nodes that are close or eliminating edges where we can maintain the characteristics of the network, while maintaining the most important topological and hydraulic properties of the network. Finally, there are quality assurance protocols that are related to the verification of the connectivity of primary features and their associated potential isolates, as well as the consideration and verification of consistency in the different graph representations of the features' physical infrastructure characteristics.

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Algorithm 1 Critical-Component Identification and Resilience Augmentation (CCIRA)

Require: Weighted network graph $G = (V, E, W)$ with capacities c_e and costs κ_e on edges, demand set $D \subseteq V$, sources $S \subseteq V$, time horizon T, budget B, candidate augmentations $\bar{E}$.
Ensure: Ranked critical components $\mathcal{C}$ and augmentation set $E^\star$.

Construct A and $L = D - A$ from G.
Compute centrality vectors C_D, C_B, C_C and eigenvector centrality x.
Compute baseline algebraic connectivity $\lambda_2(L)$ and baseline resilience R_0.
for $k = 1$ to K **do** ▷ failure scenarios
    Select failure set F_k (random or targeted by C_B).
    Form perturbed graph $G_k \leftarrow G \setminus F_k$ and evaluate service $Q_k(t)$ by a feasible flow satisfying $0 \le f_e \le c_e$.
    Compute scenario resilience $R_k \leftarrow \frac{1}{T}\int_0^T \frac{Q_k(t)}{Q_0}\, dt$.
end for
Rank components by a composite criticality score $\phi(v) = \alpha\, \tilde{C}_B(v) + \beta\, \tilde{C}_D(v) + \gamma\,(1 - \tilde{x}(v))$ and set $\mathcal{C}$ as the top-m components.
Initialize $E^\star \leftarrow \emptyset$, remaining budget $B_r \leftarrow B$.
while $B_r > 0$ and $\bar{E} \neq \emptyset$ **do**
    for all $e \in \bar{E}$ with $\kappa_e \le B_r$ **do**
        Estimate marginal gain $\Delta(e) \leftarrow R(G + e) - R(G)$ (or a surrogate using λ_2 and shortest paths).
    end for
    Choose $e^\star \leftarrow \text{argmax}\Delta(e)$ and set $E^\star \leftarrow E^\star \cup \{e^\star\}$.
    Update $B_r \leftarrow B_r - \kappa_{e^\star}$ and $\bar{E} \leftarrow \bar{E} \setminus \{e^\star\}$.
end while
return $(\mathcal{C}, E^\star)$.

Table 3. Vulnerability Assessment Framework Components.

Assessment Level	Analysis Methods	Key Indicators	Output Metrics
Component-Level	Node removal impact	Centrality scores	Criticality index
Path-Level	Flow disruption analysis	Alternative route availability	Redundancy factor
System-Level	Cascading failure simulation	Network fragmentation	Resilience score
Scenario-Based	Monte Carlo simulation	Service loss probability	Risk assessment

Vulnerability assessment uses systematic failure simulation to measure how resilient the network is against various disruptive events. Random failure simulations examine how a system responds to a random series of component failures that represent normal wear and tear, equipment design failure, or unintentional damage. Targeted failure simulation studies worst-case scenarios in which an enemy or natural disaster directly damages the highest-priority network component (e.g., a pump station or the pipelines feeding that pump station). Cascading failure models capture the result of one or more network components failing and the resulting disruption from interdependencies that reveal vulnerabilities hidden from individual-component simulation.

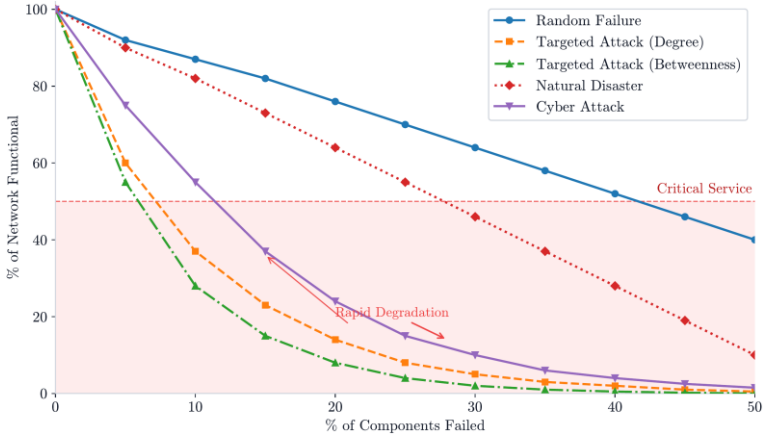


Figure 3. Network Performance under Different Failure Scenarios Cascading Failure Analysis.

The simulation framework was capable of modelling realistically operating constraints such as hydraulic (flow rates, pressure, etc) requirements and user demand patterns, aggregate service level agreements, etc. Not only were the performance measures based on connection, they also measured service quality performance measures (pressure, flow rate, service delivery reliability). The period of time that any system would be out of operation was not only the "response day" on which the outage was felt, but also included the number of all of the "structural systemic outages" for at least one

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full operational cycle, to take into account priorities for backfilling, restoration, and recovery. The probabilistic approach also included uncertainty in failure rates of the system components, fluctuations in system demand, and fluctuations in the performance of the components in operation in the surrounding environment.

Case Study Application

Lastly, to illustrate the applicability of the framework, a real-world water distribution network in a medium-sized urban area with over 500,000 people is analysed, with the aim of illustrating the practical relevance of the proposed framework. It consists of 3247 nodes (treatment plants, storage reservoirs, pumping stations and demand plants) and 4156 pipes ranging from 100 to 1200 mm in diameter. The system mirrors the natural development of the urban infrastructure of the old urban centre and the new suburban extensions.

The topological analysis shows that it exhibits a scale-free network structure, consisting of a large minority of low-degree nodes and a small minority of hub nodes with numerous links. This diversity helps the network to run more efficiently, but at the cost of higher risks of selective failure of key nodes in the network. The local redundancy (the mean value of the clustering coefficient) is found to be 0.23, which is a moderate level, with higher levels of clustering in the aged parts of the cities compared to the newer extensions. Centrality measures highlight transmission mains and large pumping stations as being critical bridging elements, and demand nodes connected with large populations have high eigenvector centrality and closeness.

The network is very robust, as shown by simulation results where, even when random failures occur up to 15% of the nodes, only about 90% connectivity is lost. On the other hand, if a few nodes are removed from the network that are highly central, the service coverage drops to approximately 62% and the network becomes highly fragmented. Results highlight the need to preserve important components and to make nodes that are central salient.

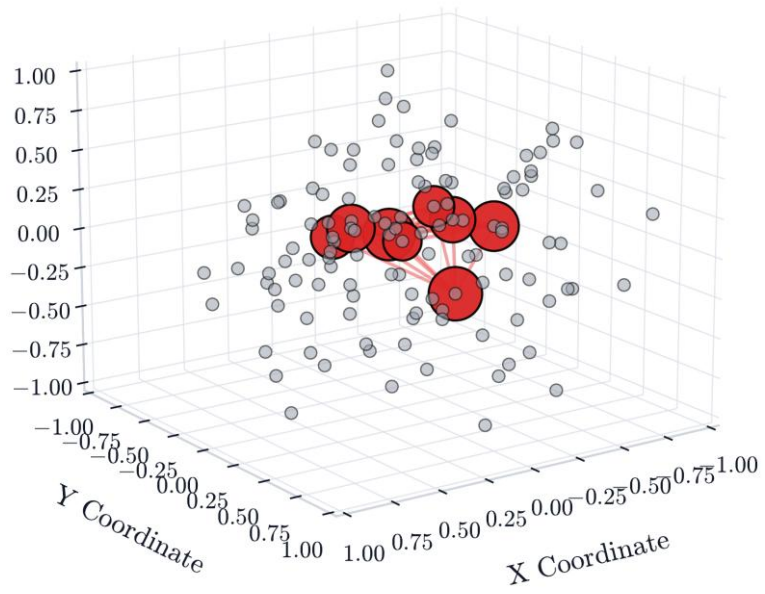


Figure 4. 3D Water Distribution Network Topology Critical Components Highlighted in Red

Operational vulnerabilities can be explored using graph analysis and hydraulic simulation. In times of peak consumption, there are multiple paths through the network that are congested. Some courses are capacity-bound. This introduces bottlenecks or thresholds, such that if a component fails in that path, then the provision of the service would be significantly affected by that failure of the component. The process of searching for critical points in the hydraulic system might not be identical to the process of searching for critical nodes in the topology and may become a convenient method of referring to specific capacity additions along it as required routes to augment normal operating capacities and backup response. Hydraulic analysis also provided zone pressure analysis, which is subject to service degradation due to high demand events and pump failure. This information will help to identify the strategic location of backup systems and pressure management devices.

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Table 4. Case Study Network Performance Metrics.

Performance Indicator	Baseline Value	After 5% Random Failure	After 5% Targeted Attack	Optimized Network
Network Connectivity	100%	94.3%	71.2%	96.8%
Average Path Length	5.7	6.2	8.9	5.3
Service Coverage	100%	92.1%	68.4%	95.2%
Redundancy Index	1.28	1.19	0.87	1.64
Resilience Score	0.72	0.65	0.41	0.89

Optimization Strategies

Optimization Problem

$$\max_{E' \subseteq \bar{E}} R(G + E') \tag{14}$$

$$\text{s.t.} \sum_{e \in E'} \text{cost}(e) \leq B. \tag{15}$$

Based on the results of the vulnerability assessment, this study develops and discusses a number of approaches to enhancing network resilience by making incremental investments in infrastructure elements and by implementing operational modifications. The thinking further provides a method of making a serious decision to arrive at an optimum allocation by taking into consideration the costs involved in the intervention as well as the measures of resilience, since the available resources/funds to develop an infrastructure will never be limitless. During the multi-objective optimization process, we are able to identify trade-off solutions that offer the greatest improvements to resilience within the budget constraint, while considering the intervening factors of intervention complexity, disruption to ongoing services, and ongoing staffing and financial costs of adherence to the intervention.

One of the key principal approaches to improving resilience is the strategic addition of edges, effectively creating alternate paths between nodes, which increases redundancy in the network and reduces vulnerability to single node or component failures. As part of the proposed optimization algorithm, we evaluate new connections based on their contribution to reducing the average path length, improving algebraic connectivity, and allowing alternative flows to critical nodes in the case of component failure. The topological advantages of adding additional edges to the network are also important, and so is making the added edges more easily constructed (e.g., right-of-way) or more logical in terms of what the original edges represent. Within simulation, only 2 % more of the edges were added to connect to a chosen group of neighboring and alternative nodes and the resilience score was estimated to be increased by 23percent variability.

Upon the results of centrality analysis, the network is hardened by directing resources at protecting and reinforcing the critical elements of the network. Hardening strategies consisted of protection from the effects of physical hazards, backup power supply for electrical outages, improved monitoring and detection of emergent problems, and response protocols designed to limit downtime. Cost-benefit analysis indicates that hardening the top 10% of critical nodes provides resilience enhancements equal to or superior to less costly system upgrades.

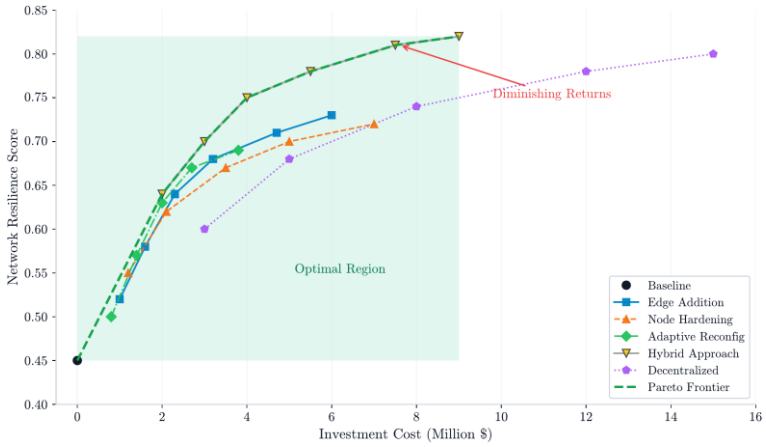


Figure 5. Multi-Objective Optimization Results Cost-Resilience Trade-off Analysis.

Adaptive network reconfiguration is a new use of existing infrastructure which employs dynamic operational techniques to respond to changing conditions and emerging threats. Smart valve systems are examples of adaptive reconfigurators that can routinely and in real-time re-route flow around damaged components. Variable speed pumps enable flow rates to adjust as needed to maintain service pressure during conditions that disrupt normal network flow. Graph-based control algorithms enable adaptive reconfiguration of network operations by synchronizing and optimizing routes for alternate network reconfigurations that minimize service interruptions while respecting hydraulic limitations on pipe and valve flows (upstream and downstream). When implemented, adaptive reconfigurations can improve the effective resilience of

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networks by 30-40% without the need for costly capital improvements or replacements. The decentralized structure is a brand-new network design philosophy and architecture. Decentralized systems change the paradigm of a centralized treatment and distribution system to distributed systems that provide treatment in local territories, predominantly working semi-autonomously. Graph partitioning algorithms can be used by planners to discover the inherent community structure of networks to reveal potential independent groups or districts that might be useful in times of need. Each potential district is a separate entity, with local storage, backup treatment capability, and interconnections that can share raw water sources under normal conditions and can isolate contaminants in the event of system contamination or a threat to individual districts or regions of infrastructure. The investment cost of the architecture is high for a network, but these districts offer higher resilience to large-scale cascading failures and system disruptions. All optimization strategies are improved with real-time and predictive analytics for continuous reporting about the network condition and emerging vulnerabilities. Graph-based anomaly detection uses algorithms to detect abnormal flow patterns or pressure variations that may indicate a new problem. Machine learning (ML) models ingest a history of past failures to predict reliability and to identify components to maintain preventively prior to failure. Predictive scoring changes infrastructure orientation from reactive to resiliency optimization: instead of reacting to failures and being involved in any failure-inducing network state before a failure, it requires less recovery time to increase resiliency without a failure occurring in the network.

Results and Discussion

Beyond qualitative observations, the improved mathematical formulation enables a quantitative interpretation of the improvements in resilience. In particular, increases in algebraic connectivity λ 2 were consistently associated with decreases in network fragmentation under targeted failures in line with theoretical predictions from spectral graph theory. Interventions based on centrality were nonlinear, with disproportionate gains in overall service continuity when protecting a small subset of high-betweenness nodes. Furthermore, flow-based analysis revealed that the robustness of the topology is not enough to make the system resilient enough for operation. In some of the scenarios, there was a high connectivity though the service was lost because of capacity-constrained bottlenecks. This result indicates that hydraulic and flow constraints are important to incorporate into the graph-based vulnerability assessment. Topological, spectral and flow metrics provide a more holistic description of infrastructure performance when working under stress.

Contact with urban systems analysis brings some interesting evidence of the relation between the topology of the network and the resilience of the infrastructure. It shows the usefulness of graph theory techniques for achieving a better understanding of urban utility systems, and their application to the future enhancement of these systems. The qualitative results confirm that traditional design paradigms can only offer functionality and design networks intentionally, and networks have unintended vulnerabilities that can be revealed by any systematic, graph-based assessment. One would be that the discovery of such vulnerabilities would yield actionable intelligence to infrastructure managers and urban planners who are having difficulty assessing the long-term durability of such systems and are struggling to find resources that can be applied to them, but are always insufficient. Previous statistical studies of different infrastructure networks have shown that there are patterns that cut across geography and also across technologies. For example, networks are empirically scale-free, with power-law degree distributions with values or exponents in the range of 2.1 - 2.7. And they emerge as a consequence of the process of preferential attachment as networks are being built. This general fact will be very important for resilience planning – namely that, regardless of the details of how a given network works, it is more efficient to protect the hub nodes than the peripheral ones. Given the small-world properties of the nodes, such as high clustering coefficient and short average path length, it is also natural to assume that infrastructure networks will grow in terms of their own structures, paying attention to local redundancy and global efficiency.

Empirical studies of the graph measures and degrees of operational performance prove the previously theorized link between networks with higher algebraic connectivity and greater resilience to random failures, serving more customers than networks with lower algebraic connectivity even when component failures occur. The Betweenness centrality showed a clear non-linear relationship with the criticality of loss in terms of failure effects, with sharp rises at certain critical values. Its quantitative nature allows for straightforward modelling of network risk based on readily-computable graph measures and hence offers the potential for a fast screening of infrastructure systems for resilience.

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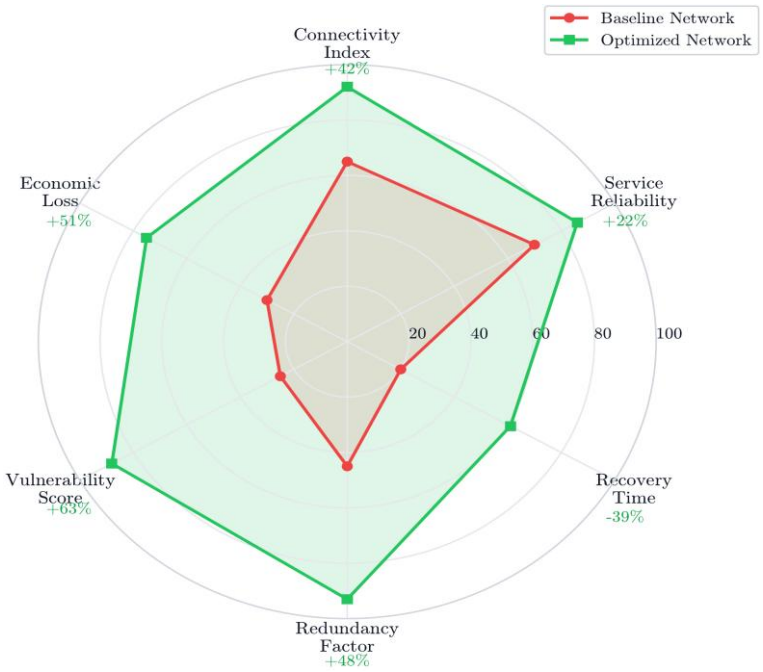


Figure 6. Full Resilience Enhancement Results Radar Chart Comparison.

A comparison of optimization strategies shows that the application of a combination of resilience mechanisms that implements several strategies simultaneously, leads to more positive results than those implementing a single strategy. The multi-vulnerability modes are synergistic when organizations add edges for topological redundancy, harden nodes for critical component protection, and reconfigure to deal with dynamic disruptions. The cost effectiveness analysis illustrates that strategies are being achieved at 85% of the benefit of full resilience enhancement at 40% of the cost allowing for strategies to be targeted.

management. Graph visualizations of a static structure can point out potential weak points but do not consider dynamic variables such as demands, maintenance patterns, and seasonal variability of the systems that could be stressing the systems and affecting resilience. Dynamic graph-based models using temporal edges and node properties evolution are used to solve tasks like resilience evaluation in real-world scenarios and dynamic temporal optimization strategies. Alternative planning and maintenance strategies based on time of year and demand, as well as emergency response preparations before and during extreme weather events, all offer opportunities to steer an organization toward improved resilience without any long-term changes to infrastructure. Sensitivity Analysis is the study of how changes in the input parameters and assumptions about the model affect the recommendations for optimization to exercise trust in the output from models based on a graph. The results indicate that it is stable when the estimates, probabilities and costs for demand change also moderately. Thus, graph-based recommendations are robust and trusted even if they are based on uncertain inputs.

Large-scale scenarios and black swan events are rarely included in normal modeling paradigms and often need adaptive management approaches to deal with challenging, uncertain and unexpected unknowns. The graph-theoretic framework integrates the scenario planning and the stress testing to identify the critical points and threshold effects which can lead to large changes in behavior that affect the overall network function.

Implementation Challenges

Although of theoretical and empirical value, graph-based infrastructure analysis suffers from several practical challenges that need to be solved for its successful implementation in real operational environments. The main challenges of network analysis are the problems of data quality and availability, because an analyst needs to know very specific information about infrastructure elements and their links, as well as details about the performance of these. Most utilities don't have complete digital documentation of their systems because they don't have records of infrastructure built before today's documentation requirements went into effect. There is enormous expense and technology involved in digitizing legacy records, verifying all network configurations and maintaining all databases on an ongoing basis.

The increased complexity of the infrastructure networks with hydraulic limits and water quality measures, and the growing amount of real time data, poses a new challenge because of the computations. While the basic graph metrics are calculated efficiently and are scalable for large networks, the complete vulnerability analysis, including simulation of failures and cascades and multi-objective optimization is costly for large networks. To cope with the computational demands, approximation algorithms, parallel processing and hierarchical modeling approaches have been devised for the field, but scientists are still challenged to balance the precision of the model with the computational resources. Institutional barriers are more difficult than technical challenges because implementing graph-based optimization systems

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requires organizations to bring together disparate departments, agencies, and stakeholder groups. The traditional organizational structures in utilities and government agencies are not well suited to the integrated system approach that graph analysis provides. The different offices do not work with each other, and priorities are different or staff disagree with the action. As such it has become very difficult to implement the recommendations made by the optimization for greater benefit. To make analytical conclusions work in practice, organizations need to apply network thinking and establish collaborative governance systems.

Network planning is a special form of graph analysis, coupled with a current infrastructure management system and the associated processes and routines, when the coupling is assumed in practice. There are both organizational and technical barriers to moving between particular graph metrics and to and toward current practices of infrastructure management. They may be opposed because of the old software systems used to implement planning and operational performance, the old types of operational routines and the old operational professional standards. A company can be successfully being managed to own change in regard to disorders of accepted cognitive human habits in adopting the new suggested ways to do things by training personnel and through stratified implementation with nominal demonstration projects of the value of any graph based approach. The complex dimension graph metrics used to measure, and communicate in the usable and effective translational decision support tools that constitute the user interfaces fill the gap between theory and practice. New technologies are being developed at a rate far beyond the capacity of regulatory and standards regimes to match, leading to uncertainty about how to establish appropriate infrastructure and working practices. Current design requirements focus on individual component aspects rather than the overall system resilience, which can be a barrier to uptake of network-optimized solutions. Organizations need to develop a persistent evidence-based advocacy to change regulations to allow design standards to be revised and new design standards to be generated that include tenets of network resiliency. The organizations are conducting graph-based optimization demonstrations to demonstrate how network optimization can drive regulatory change.

Future Directions

Engineering infrastructure has a long history with graph theory and is a growing emerging discipline with the increasing computational power, increasing access to data and an emerging theoretical knowledge base. Classical graph-theoretic methods for assessing the composite resilience of infrastructure can be improved in several ways. Artificial intelligence then joins with machine learning and systems of graph analysis to provide an opportunity to create engineering innovations. The advantage of the addition to the GNN is that complex infrastructure patterns are too expensive to visualize with a more traditional methodology, leading to a better policy for vulnerability detection and optimization. Reinforcement learning can produce network topologies and functionality that a human programmer would have had difficulty conceiving. One of the fundamental aspects of research is the analysis framework of infrastructure system interconnections, which should be addressed at once. Cities are now hierarchical systems linking water supply systems to energy grids and transport systems and communication networks. The explicit interdependencies of the multilayer network models lead to new failure modes that do not exist when performing vulnerability analysis with the standard single-layer network analysis. The study would need two important elements: 1) standard models of multiple infrastructures; and 2) any standards for data sharing that utility operators may have. The edge computing systems and the IoT sensors would consider real-time analysis to get adaptive management systems that inject graph analysis into the system in real time to measure changes in the network. The on-board digital twin concepts are based on measuring the flow rates and pressure values, water quality parameters, and component conditions and all these are working simultaneously with the physical infrastructure to replicate its functioning to the finest extent over time. The digital twin system would then be processed through graph algorithms to detect anomalies, predict failures and optimize operations, in effect transforming non-adaptive infrastructure to a self-healing adaptive system.

One of the key requirements to adapt to climate change is the introduction of the second infrastructure resilience method. The methods must take into account the evolution of threats posed by the environment and weather patterns. For a graph-based assessment, Mullen Dore estimates that an optimized vulnerability review should incorporate climate forecasts, along with the probability of extreme events and their prolonged transformations. There is growing uncertainty about future conditions, and this requires the development of effective optimization techniques that can deal with multiple future scenarios rather than single scenarios. The only way to ensure a successful wide-scale implementation of graph analysis methods and techniques in utility organizations is to ensure that these tools are offered to every utility, irrespective of the level of operations and the resources available to the utility. It's just a matter of looking at the number of open-source software platforms available, the number of analytic services available through cloud computing, and common data formats to realize that there are entry barriers and then smaller utilities can make use of more sophisticated analytic tools. Education will be shared to help develop and support resilient systems, and educate the next generation of infrastructure workers on network thinking and graph analysis.

The issues of social equity have to be somehow accommodated in the framework. All this should take care of the benefits trickling down to each individual in society. An optimization technique used in resilience that is based on graphs. The two methodologies in use only address technical performance and indicators on the system level and thus mask disproportionate impacts on various social groups. If you're an engineer, it is important to come up with metrics for graphs and to figure out what they should be optimized for. We need a level playing field (to ensure human dignity, by addressing

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the needs of the vulnerable, environmental justice issues, and building community resiliency to sustainable and resilient infrastructure systems).

Conclusions

The study proves mathematically that abstract graph-theoretic concepts can be turned into useful metrics to inform infrastructure planning and management. The resilience assessment is no longer a descriptive process, but a reproducible, quantifiable process of the spectral measures, optimization formulations of the analytical workflow, and the representations based on the embedding matrix. Results show that modest, targeted measures—driven by centrality and spectral metrics—can yield resilience improvements on par with large-scale infrastructure upgrades. Utilities with limited resources face important implications from this finding. The framework will be extended in future work to multi-layer and time-varying networks, to enable a more thorough investigation of interdependencies and adaptive control mechanisms in complex urban systems. The difficult investigation of the application of graph ideas to the study and intensification of the functionality of water and alertness networks shows the possibility of reorganizing the analysis of major infrastructure irritations by the use of mathematical inquiries. It has been proved that graph-theoretic techniques are among the useful tools to help us in investigating the flaws in communities and identifying the necessary additions to design the version strategies that are adequate to counter the device nimbleness under realistic resource constraints. A combination of topological evaluation and hydraulic modeling and consequences of operating data and failure simulation in a global framework is used to assess infrastructure that stresses both structural and functional overall performance criteria of the population.

The application exploration and case-based empirical investigations validate network topology and prove the theoretical assumptions of the relationship linking network topology and versatility. A graph can generate sufficient improvement in the fortification of the intervention system entirely based on graph analysis. Generally the size of shared styles seen in the organization of infrastructure, the scale-free diploma circulation and small overseas dwellings, gives the feel of flexibility plan in most situations and parameters. The development of multi-reason version techniques that provide stability, cost, overall performance and utility of action displays reflects the economic price of graph-intensive techniques of organizing the reality structure.

The main implications proposed here are: the need to access the key nodes of the hub to provide intentional redundancy and the implementation of dynamic patterns of activity, taking advantage of the plasticity of the community. The literature indicates that for a network, the resilience gains that can be achieved from a relatively small investment in specific network upgrade investments can be significantly greater than those of traditional off-the-shelf approaches that try to improve resilience across the network in a uniform way. In the future, predictive analytics and real-time tracking of events with the help of graph-based management algorithms will lead to almost self-optimising structures of the infrastructure which can respond and react to perturbations automatically. The large-scale problems of statistical quality, computational complexity, institutional constraints and regulatory frames must be resolved in the application of graph-based techniques, especially optimization techniques. Success factors are technical solutions as well as organizational exchange, participation and capacity building of stakeholders. Development of clean-to-use equipment, common methodologies and sample applications will likely be the key to further large-scale adoption of graph-theoretic methods in the water sector and software sector.

A higher degree of sociological evolution made its new speech do without such a strange work, but also without a strange notion, and even without an insight into the control of all types of feeling forms (e.g., very much expected mineral). The first part of the list includes a scheme of research and alertness, integration of Artificial Intelligence, Multi-layer Community modeling, Real-Time analytics and Climate Variability. Graph theory provides a useful mathematical context both for designing and reacting to the malleable patterns of infrastructural life that will be able to sustain the life of the city into an uncertain future, while cyber-attacks are a growing threat that sets demand conditions. An alternative graph-theoretic indicator of how well infrastructure is flexible is not one that can be mathematically measured but, perhaps, a more concrete one: making services more reliable, disaster-proof, and network-healthy. This study can be classified into smart analytic actions, pragmatic adjustment steps, and data sources that provide useful information to model the graph concept. It also accommodates the shape of published work at this level of study, practicality and policy, which is already guaranteed to provide even more elastic, eternal and just urban frameworks.

The paper shows that in arithmetic, engineering, computer technological skill and concrete planning are inter-disciplinary - an affidavit to the current problem of infrastructure structures. The challenges of modern projects in building infrastructure are solved through the most "expansive" way of construction – crossing traditional borders, uniting points of view and with the help of scientific elements in a non-conformable decision making process. Language and analysis tool – a common language and a common analysis tool – graph theory – that would enable such convergence, could be the language and platform for analysis that stakeholders would weave into a more mundane agenda – reliable infrastructure that serves humans fairly, reliably.

Conflict of interest. Nil

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