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*Original article*

# Solving Fractional Schrödinger Equations via Homotopy Perturbation Sumudu Transform Method

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**Abstract**
We propose and analyze a technique that combines the homotopy perturbation method and the Sumudu transform method. This method is called the homotopy perturbation Sumudu transform method. We use this method to solve the generalized time-space fractional Schrödinger equation. The fractional derivative is described in the Caputo sense. The proposed scheme finds the solution without any discretization, transformation, or restrictive assumptions. We present several examples to verify the reliability and efficiency of the proposed technique.
**Keywords.** Caputo Derivative, Homotopy Perturbation Method, Sumudu Transform Method, Schrödinger Equation.

## Introduction

In recent years, considerable interest in fractional differential equations has been stimulated due to their numerous applications in the areas of nonlinear science [4], many important phenomena [20], engineering and physics [18], dielectric polarization [23], and quantitative finance [15]. To find explicit solutions of linear and nonlinear fractional differential equations, many powerful methods have been used, such as the homotopy perturbation method [7,19, 24] and the Fourier transform method [17].
The Sumudu transform method [8], the domain decomposition method [10, 21], the variational iteration method [25], the homotopy analysis method [1], the fractional complex transform [5], and homotopy perturbation Sumudu transform method [13, 22].
In this paper, we investigate the application of the homotopy perturbation Sumudu transform method (HPSTM) for solving the generalized Time-space fractional Schrödinger equation with variable coefficients [11, 21]:

$$i\ D_t^{\alpha} u(x,t) + a\frac{\partial^{2\beta}}{\partial x^{2\beta}} u(x,t) + v(x)u(x,t) + \gamma u(x,t)|u|^2 = 0, \qquad (1)$$

where $t > 0,\ 0 < \alpha, \beta \le 1$ with initial conditions

$$u(x,0) = u_0(x,t), \qquad (2)$$

Where $u = u(x,t)$ is an unknown function, $v(x)$ is the trapping potential, $0 < \alpha, \beta \le 1$ are parameters describing the order of the fractional Jumaries derivatives [12], and $a, \gamma$ are real constants, respectively. If we select $\alpha = \beta = 1, v(x) = 0$ this equation turns into the famous nonlinear Schrödinger equation in optical fiber [3, 9, 16]. In this paper, notice that (1) is a complex differential equation with a complex modulus term $|u|^2$ as we all know, a complex function $u(\xi)$ can be written as $u(\xi)e^{i\theta(\xi)}$ where $c(\xi)$ and $\theta(\xi)$ are real functions; notice that $|u(\xi)|^2 = |c(\xi)|^2$, assume that $\lim_{n\to\infty} |\tilde{u}|^2 = |u_0|^2$, we get the HPSTM for Equation (1).

### *Basic definitions of fractional calculus*

In this section, we present the basic definitions and properties of the fractional calculus theory, which are used further in this paper.

**Definition 1.** The Riemann-Liouville fractional integral operator of order $\alpha > 0,$ of a function $f \in C_{\sigma},\ \sigma \ge -1$ is defined by [20] as follows:

$$J^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\xi)^{\alpha-1} f(\xi)\, d\xi, \qquad (3)$$
$$J^0 f(t) = f(t).$$

**Definition 2.** The fractional derivative of $f(t)$ in the Caputo sense is defined by [20] as follows:

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$$D_t^{\alpha} f(t) = \begin{cases} \dfrac{1}{\Gamma(n-\alpha)} \displaystyle\int_0^t (t-\zeta)^{n-\alpha-1} f^{(n)}(\zeta)\ d(\zeta), & n-1 < \alpha \le n, \\ D_t^{\alpha} f(t), & \alpha = n, \end{cases} \tag{4}$$

Also one has the following properties:

$D^{\alpha} C = 0,$ ( $C$ is constant ),

$$D^{\alpha} x^{\gamma} = \begin{cases} \dfrac{\Gamma(\gamma+1)}{\Gamma(\gamma-\alpha+1)} x^{\gamma-\alpha}, & \gamma > \alpha - 1, \\ 0, & \gamma \le \alpha - 1, \end{cases}$$

$$J^{\alpha} D^{\alpha} f(x) = f(x) - \sum_{k=0}^{n-1} f^{(k)}(0^{+}) \frac{x^k}{k!}, \quad n-1 < \alpha \le n,$$

$$D^{\alpha} J^{\alpha} f(x) = f(x).$$

**Definition 3.** The Mittag-Leffler function, which is a generalization of the exponential function, is defined by [14] as follows:

$$E_{\alpha}(t) = \sum_{j=0}^{\infty} \frac{t^j}{\Gamma(\alpha j + 1)}, \quad \alpha > 0, \quad t \in C,$$

$$E_{\alpha,\beta}(t) = \sum_{j=0}^{\infty} \frac{t^j}{\Gamma(\alpha j + \beta)}, \quad \alpha, \beta > 0, \quad t \in C.$$

Some special cases of the Mittag-Leffler function are as follows:

$E_1(t) = e^t,$

$E_{\alpha,1}(t) = E_{\alpha}(t).$

**Definition 4.** The Sumudu transform is defined by [2] over the set of functions:

$$A = \left\{ f(t) \ / \ \exists\ M,\ \tau_1,\ \tau_2 > 0,\ |f(t)| < M\ e^{\frac{|t|}{t_j}}\ \ if\ \ t \in (-1)^j \times [0, \infty) \right\},$$

by the following formula:

$$\bar{f}(u) = S[f(t)] = \int_0^{\infty} f(ut)\ e^{-t}\ dt,\ u \in (\tau_1, \tau_2). \tag{5}$$

Some special properties of the Sumudu transform are by [2] as follows:

$S[1] = 1,$

$$S\left[\frac{t^m}{\Gamma(m+1)}\right] = U^m.$$

**Definition 5.** The Sumudu transform of the Caputo fractional derivative is defined by [8] as follows:

$$S\left[D_t^{\alpha} f(x,t)\right] = U^{-\alpha} S\left[f(x,t)\right] - \sum_{k=0}^{m-1} U^{-\alpha+k} f^{(k)}(0+), \quad m-1 < \alpha \le m. \tag{6}$$

***Homotopy Perturbation Sumudu Transform Method (HPSTM)***

To illustrate the basic concept of the technique, we consider the following general fractional partial differential equation with the initial condition of the form:

$$D_t^{\alpha} u(x,t) + L\,u(x,t) + N\,u(x,t) = f(x,t), \tag{7}$$

With $m - 1 < \alpha \le m$ and subject to the initial condition

$$\frac{\partial^s}{\partial t^s} u(x,0) = u^{(s)}(x,0) = f_s(x), \quad s = 0,1,\ldots,n-1, \tag{8}$$

Where $D_t^{\alpha} u(x,t)$ is the Caputo fractional derivative of the function $u(x,t)$, $f(x,t)$ is the source term, $L$ is the linear differential operator and $N$ is the general nonlinear differential operator.

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Applying the Sumudu transform (denoted in this paper by $S$ ) on both sides of Equation (7), we get

$$S\left[D_t^{\alpha} u(x,t)\right]+S\left[L\,u(x,t)\right]+S\left[N\,u(x,t)\right]=S\left[f(x,t)\right]. \quad (9)$$

Using the differentiation property of the Sumudu transform and the initial conditions in Equation (8), we have

$$S\left[u(x,t)\right]=\sum_{k=0}^{m-1} U^k\ u^{(k)}(x,0)+U^{\alpha} S\left[f(x,t)-Lu(x,t)-Nu(x,t)\right]. \quad (10)$$

Operating with the Sumudu inverse on both sides of Equation (10) gives

$$u(x,t)=F(x,t)+S^{-1}\left[U^{\alpha} S\left[f(x,t)-Lu(x,t)-Nu(x,t)\right]\right], \quad (11)$$

Where $F(x,t)$ represent the prescribed initial conditions. Now we apply the HPM.

$$u(x,t)=\sum_{n=0}^{\infty} p^n u_n(x,t), \quad (12)$$

And the nonlinear term can be decomposed as

$$Nu(x,t)=\sum_{n=0}^{\infty} p^n A_n, \quad (13)$$

For some Adomian's polynomials $A_n$ that are given by [6]

$$A_n=\frac{1}{n!}\frac{d^n}{dp^n}\left[N\left(\sum_{j=0}^{\infty} p^j u_j(x,t)\right)\right]_{p=0}, \qquad n=0,1,2,\ldots \quad (14)$$

Substituting Equation (12) and Equation (13) in Equation (11), we get

$$\sum_{n=0}^{\infty} p^n u_n(x,t)=F(x,t)+S^{-1}\left[U^{\alpha} S\left[f(x,t)-L\left(\sum_{n=0}^{\infty} p^n u_n(x,t)\right)-\sum_{n=0}^{\infty} p^n A_n\right]\right]. \quad (15)$$

Equating the terms with identical powers of $p$ , we can obtain a series of equations as the follows:

$$\begin{aligned}
&p^0: u_0(x,t)=F(x,t),\\
&p^1: u_1(x,t)=S^{-1}\left[U^{\alpha} S\left[f(x,t)-Lu_0(x,t)-A_0\right]\right],\\
&p^2: u_2(x,t)=S^{-1}\left[U^{\alpha} S\left[f(x,t)-Lu_1(x,t)-A_1\right]\right], \quad (16)\\
&\vdots\\
&p^n: u_n(x,t)=S^{-1}\left[U^{\alpha} S\left[f(x,t)-Lu_{n-1}(x,t)-A_{n-1}\right]\right],
\end{aligned}$$

Proceeding in the same manner, the rest of the components $u_n(x,t)$ can be completely found and the series solution is thus entirely determined. We approximate the analytical solution $u(x,t)$ by truncated series as:

$$u(x,t)=\lim_{p\to 1}\sum_{n=0}^{\infty} p^n u_n(x,t). \quad (17)$$

***Applications***

In this section, we apply the HPSTM developed in Section 3 to solve the Generalized Time-space Fractional Schrödinger Equation with variable coefficients. Numerical results reveal that the HPSTM is easy to implement and significantly reduces the computational effort while a tangible maintaining a very high level of accuracy. For comparison, we evaluate the same examples presented in [ 10, 11, 21, 25, 26].

**Example 1**

We first consider the time-fractional NLS equation:

$$i\,D_t^{\alpha} u(x,t)+a\frac{\partial^2}{\partial x^2}u(x,t)+\gamma\,u(x,t)|u|^2=0, \quad (18)$$

Where $t>0,\ 0<\alpha,\beta\le 1$ with initial conditions

$$u(x,0)=A\ \mathrm{sech}(x). \quad (19)$$

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Applying the Sumudu transform on both sides of Equation (18), thus we get

$$S\left[D_t^{\alpha}u(x,t)\right]=S\left[ai\,\frac{\partial^2}{\partial x^2}u(x,t)+\gamma i\,u(x,t)\left|u_0\right|^2\right]. \tag{20}$$

Operating with the Sumudu inverse on both sides of Eq.(20) we get

$$u(x,t)=A\,\operatorname{sech}(x)+S^{-1}\left[U^{\alpha}S\left[ai\,\frac{\partial^2}{\partial x^2}u(x,t)+\gamma i\,u(x,t)\left|u_0\right|^2\right]\right]. \tag{21}$$

By applying the homotopy perturbation method we have

$$\sum_{n=0}^{\infty}p^n u_n(x,t)=A\,\operatorname{sech}(x)+S^{-1}\left[U^{\alpha}S\left[\begin{array}{l}ai\,\dfrac{\partial^2}{\partial x^2}\left(\displaystyle\sum_{n=0}^{\infty}p^n u_n(x,t)\right)\\ +\gamma i\left(\displaystyle\sum_{n=0}^{\infty}p^n u_n(x,t)\right)\left|u_0\right|^2\end{array}\right]\right]. \tag{22}$$

Comparing the coefficient of like powers of $p$, we have

$$\left.\begin{aligned}
p^0:u_0(x,t)&=A\,\operatorname{sech}(x),\\
p^1:u_1(x,t)&=A\left(\operatorname{sech}(x)-2\operatorname{sech}^3(x)\right)\frac{ai\,t^{\alpha}}{\Gamma(\alpha+1)}+\gamma A^3\operatorname{sech}^3(x)\,\frac{it^{\alpha}}{\Gamma(\alpha+1)}\\
&=A\,\operatorname{sech}(x)\frac{ai\,t^{\alpha}}{\Gamma(\alpha+1)},\qquad\left(A^2=\frac{2a}{\gamma}\right),\\
p^2:u_2(x,t)&=A\left(\operatorname{sech}(x)-2\operatorname{sech}^3(x)\right)\frac{\left(ai\,t^{\alpha}\right)^2}{\Gamma(2\alpha+1)}+2A\operatorname{sech}^3(x)\,\frac{\left(ai\,t^{\alpha}\right)^2}{\Gamma(2\alpha+1)}
\end{aligned}\right\}$$

$$\left.\begin{aligned}
&=A\,\operatorname{sech}(x)\frac{\left(ai\,t^{\alpha}\right)^2}{\Gamma(2\alpha+1)},\\
p^3:u_3(x,t)&=A\,\operatorname{sech}(x)\frac{\left(ai\,t^{\alpha}\right)^3}{\Gamma(3\alpha+1)},\\
&\vdots\\
p^n:u_n(x,t)&=A\,\operatorname{sech}(x)\frac{\left(ai\,t^{\alpha}\right)^n}{\Gamma(n\alpha+1)}.
\end{aligned}\right\} \tag{23}$$

Thus the solution of Equation (18) is given by

$$\left.\begin{aligned}
u(x,t)&=\lim_{p\to 1}\sum_{n=0}^{\infty}p^n u_n(x,t)\\
&=A\,\operatorname{sech}(x)\left(1+\frac{ai\,t^{\alpha}}{\Gamma(\alpha+1)}+\frac{\left(ai\,t^{\alpha}\right)^2}{\Gamma(2\alpha+1)}+\frac{\left(ai\,t^{\alpha}\right)^3}{\Gamma(3\alpha+1)}+\cdots\right)\\
&=A\,\operatorname{sech}(x)\sum_{n=0}^{\infty}\frac{\left(ai\,t^{\alpha}\right)^n}{\Gamma(n\alpha+1)}\\
&=A\,\operatorname{sech}(x)\,\mathrm{E}_{\alpha}\left(ai\,t^{\alpha}\right).
\end{aligned}\right\} \tag{24}$$

If we put $\alpha\to 1$ in Equation (24) or solve Equation (18) and (19) with $\alpha=1$, we obtain the exact solution

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$$u(x,t) = A\ \sec h(x)\ \mathrm{E}_{\alpha}\left(a\,i\,t^{\alpha}\right)$$
$$= \pm\sqrt{\frac{a}{\gamma}}\ \sec h(x)\ e^{a\,i\,t}.$$

Which is in full agreement with the result in Ref. [11, 26].

**Example 2**

We first consider the time-space fractional NLS equation:

$$i\ D_t^{\alpha}\,u(x,t) + \frac{1}{2}\frac{\partial^{2\beta}}{\partial x^{2\beta}}u(x,t) - u(x,t)\cos^2(x) - u(x,t)\left|u\right|^2 = 0, \tag{25}$$

Where $t > 0,\ 0 < \alpha, \beta \le 1$ with initial conditions

$$u(x,0) = \sin(x). \tag{26}$$

Applying the Sumudu transform on both sides of Equation (25), thus we get

$$S\left[D_t^{\alpha}\,u(x,t)\right] = S\left[\frac{1}{2}i\ \frac{\partial^{2\beta}}{\partial x^{2\beta}}u(x,t) - i\ u(x,t)\cos^2(x) - i\,u(x,t)\left|u_0\right|^2\right]. \tag{27}$$

Operating with the Sumudu inverse on both sides of Eq.(27) we get

$$u(x,t) = \sin(x) + S^{-1}\left[U^{\alpha}S\left[\begin{array}{c}\frac{1}{2}i\ \frac{\partial^{2\beta}}{\partial x^{2\beta}}u(x,t) - i\ u(x,t)\cos^2(x)\\ \\ -i\,u(x,t)\left|u_0\right|^2\end{array}\right]\right]. \tag{28}$$

By applying the homotopy perturbation method we have

$$\sum_{n=0}^{\infty} p^n\,u_n(x,t) = A\ \sec h(x) + S^{-1}\left[U^{\alpha}S\left[\begin{array}{l}\frac{1}{2}i\ \frac{\partial^{2\beta}}{\partial x^{2\beta}}\left(\sum_{n=0}^{\infty} p^n\,u_n(x,t)\right)\\ \\ -i\,u\left(\sum_{n=0}^{\infty} p^n\,u_n(x,t)\right)\cos^2(x)\\ \\ -i\left(\sum_{n=0}^{\infty} p^n\,u_n(x,t)\right)\left|u_0\right|^2\end{array}\right]\right]. \tag{29}$$

Comparing the coefficient of like powers of $p$, we have

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$$
\left.
\begin{aligned}
p^{0}&: u_{0}(x,t) = \sin(x),\\
p^{1}&: u_{1}(x,t) = \left(\frac{1}{2}\sin(x+\pi\beta) - \sin(x)\left(\sin^{2}(x)+\cos^{2}(x)\right)\right)\frac{it^{\alpha}}{\Gamma(\alpha+1)}\\
&\quad = \left(\frac{1}{2}\sin(x+\pi\beta) - \sin(x)\right)\frac{i\,t^{\alpha}}{\Gamma(\alpha+1)},\\
p^{2}&: u_{2}(x,t) = \left(\begin{aligned}&\frac{1}{4}\sin(x+2\pi\beta) - \frac{1}{2}\sin(x+\pi\beta)\\ &-\left(\sin^{2}(x)+\cos^{2}(x)\right)\left(\frac{1}{2}\sin(x+\pi\beta) - \sin(x)\right)\end{aligned}\right)\frac{it^{\alpha}}{\Gamma(\alpha+1)}\\
&\quad = \left(\frac{1}{4}\sin(x+2\pi\beta) - \sin(x+\pi\beta) + \sin(x)\right)\frac{\left(it^{\alpha}\right)^{2}}{\Gamma(2\alpha+1)},\\
p^{3}&: u_{3}(x,t) = \left(\begin{aligned}&\frac{1}{8}\sin(x+3\pi\beta) - \frac{3}{4}\sin(x+2\pi\beta)\\ &+\frac{3}{2}\sin(x+\pi\beta) - \sin(x)\end{aligned}\right)\frac{\left(it^{\alpha}\right)^{3}}{\Gamma(3\alpha+1)},\\
p^{4}&: u_{4}(x,t) = \left(\begin{aligned}&\frac{1}{16}\sin(x+4\pi\beta) - \frac{1}{2}\sin(x+3\pi\beta)\\ &+\frac{3}{2}\sin(x+2\pi\beta) - 2\sin(x+\pi\beta) + \sin(x)\end{aligned}\right)\frac{\left(it^{\alpha}\right)^{4}}{\Gamma(4\alpha+1)},\\
&\vdots\\
p^{n}&: u_{n}(x,t) = C_{n}(x)\frac{\left(it^{\alpha}\right)^{n}}{\Gamma(n\alpha+1)}.
\end{aligned}
\right\} \qquad (30)
$$

Where

$$C_{n}(x) = C_{n,0}(x)\sin(x) + C_{n,1}(x)\sin(x+\pi\beta) + C_{n,2}(x)\sin(x+2\pi\beta) + \cdots + C_{n,n}(x)\sin(x+n\pi\beta)$$

And where

$$C_{n,0} = (-1)^{n}, \quad n \geq 0,$$

$$C_{n,1} = \frac{1}{2}C_{n-1,0} - C_{n-1,1}, \quad n > 1,$$

$$C_{n,2} = \frac{1}{2}C_{n-1,1} - C_{n-1,2}, \quad n > 2,$$

$$\vdots$$

$$C_{n,i+1} = \frac{1}{2}C_{n-1,i} - C_{n-1,i+1}, \quad i = 0,1,2,\ldots$$

$$C_{n,n} = \frac{1}{2}C_{n-1,n-1}, \quad n \geq 1.$$

Thus the solution of Equation (25) is given by

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$$\left.\begin{aligned} u(x,t) &= \lim_{p\to 1}\sum_{n=0}^{\infty} p^n u_n(x,t) \\ &= \sum_{n=0}^{\infty} C_n(x)\frac{\left(it^{\alpha}\right)^n}{\Gamma(n\alpha+1)} \\ &= \sin\left[\frac{x^{\beta}}{\Gamma(\beta+1)}\right] Exp\left[-\frac{3}{2}\frac{it^{\alpha}}{\Gamma(\alpha+1)}\right]. \end{aligned}\right\} \quad (31)$$

If we put $\alpha, \beta \to 1$ in Equation (31) or solve Equations (25) and (26) with $\alpha = \beta = 1$, we obtain the exact solution

$$\begin{aligned} u(x,t) &= \sum_{n=0}^{\infty} C_n(x)\frac{\left(it^{\alpha}\right)^n}{\Gamma(n\alpha+1)} \\ &= \sin(x)\, e^{-\left(3/2\right)it}. \end{aligned}$$

Which is in full agreement with the result in Ref.[11, 21, 26].

Comparisons between the real part of some numerical results and the exact solution Equation (31) are summarized in (Tables 1 and 2), and the simulations for $u_4$, $u_{abs}$, and $u$ are plotted in (Figures 1 and 2), which show that HPSTM produced a rapidly convergent series.

Comparisons between the imaginary part and the exact solution Equation (31) are plotted in (Figures 3 and 4), and the simulations for $u_5$, $u_{abs}$ and $u$, show that HPSTM produced a rapidly convergent series.

***Table 1. Comparison between the real part of $u_4$ and $u$ when $\alpha = \beta = 1$.***

| $x$ | $t$ | Approximate $u_{4\,appr}$ | Exact solution | Absolute error |
|---|---|---|---|---|
| 0.2 | 0.1 | 0.1964384915 | 0.1964384884 | $3.1\times10^{-9}$ |
| 3 | 0.2 | 0.1348172358 | 0.1348170931 | $1.427\times10^{-7}$ |
| 15 | 0.3 | 0.5855572742 | 0.5855498014 | $7.4728\times10^{-6}$ |

***Table 2. Comparison between the real part of $u_4$ and $u$ when $\alpha = 0.7,\ \beta = 0.9$.***

| $x$ | $t$ | Approximate $u_{4\,appr}$ | Exact solution | Absolute error |
|---|---|---|---|---|
| 0.2 | 0.1 | 0.1989280524 | 0.2288404399 | 0.0299123875 |
| 0.2 | 0.2 | 0.1978265247 | 0.2080382267 | 0.010211702 |
| 1 | 0.3 | 0.6273816436 | 0.6355277868 | 0.0081461432 |

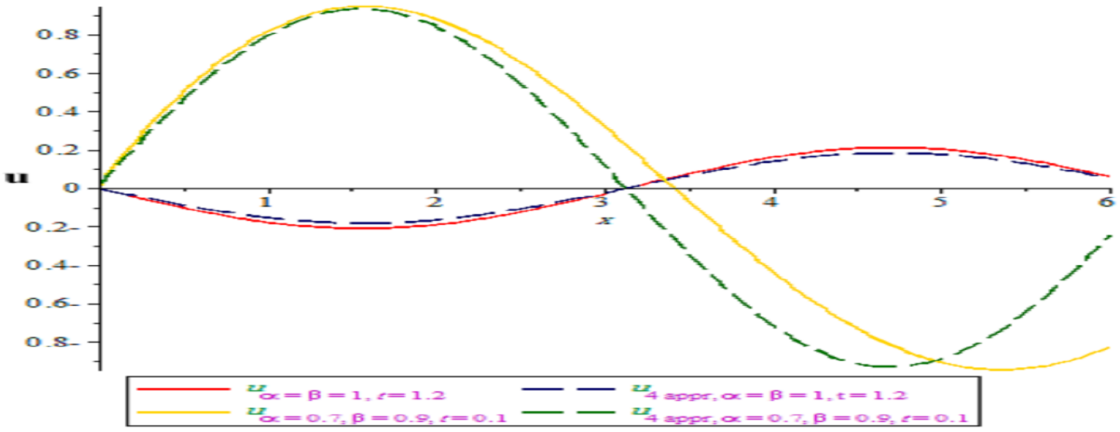



***Figure 1. Comparison between the real part of $u_4$ and the exact solution $u$.***

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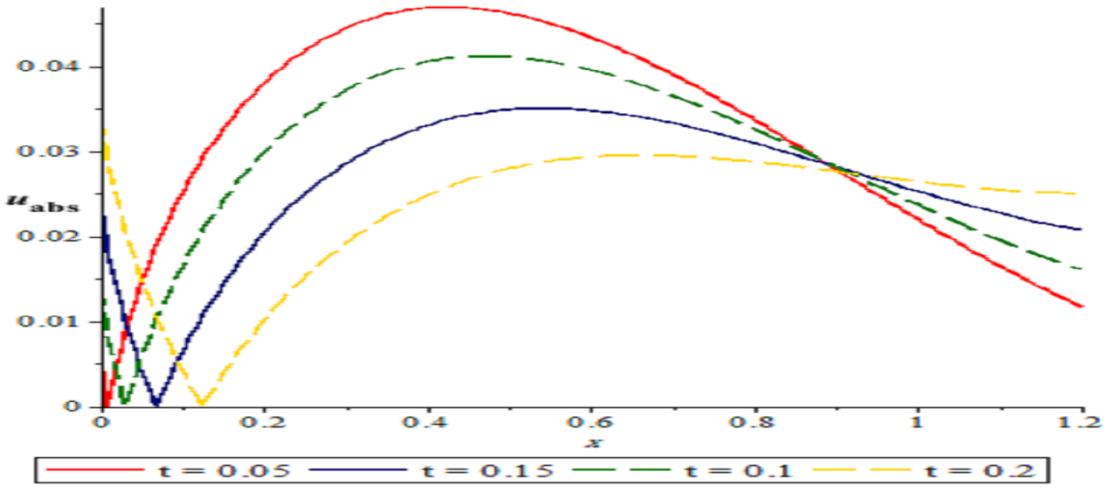



***Figure 2. Plots of the absolute error*** $u_{abs}$ ***when*** $\alpha = 0.7$ ***and*** $\beta = 0.9$ **.**

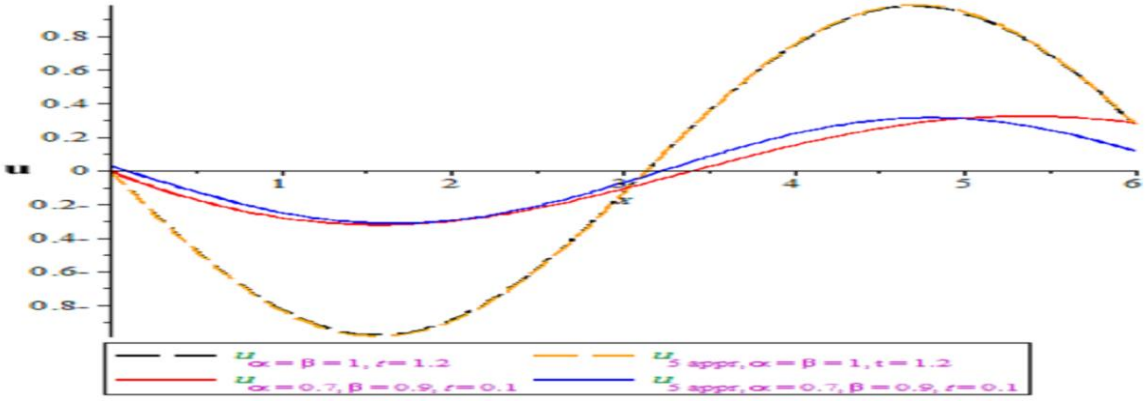



***Figure 3. Comparison between the imaginary part of*** $u_5$ ***and the exact solution*** $u$ **.**

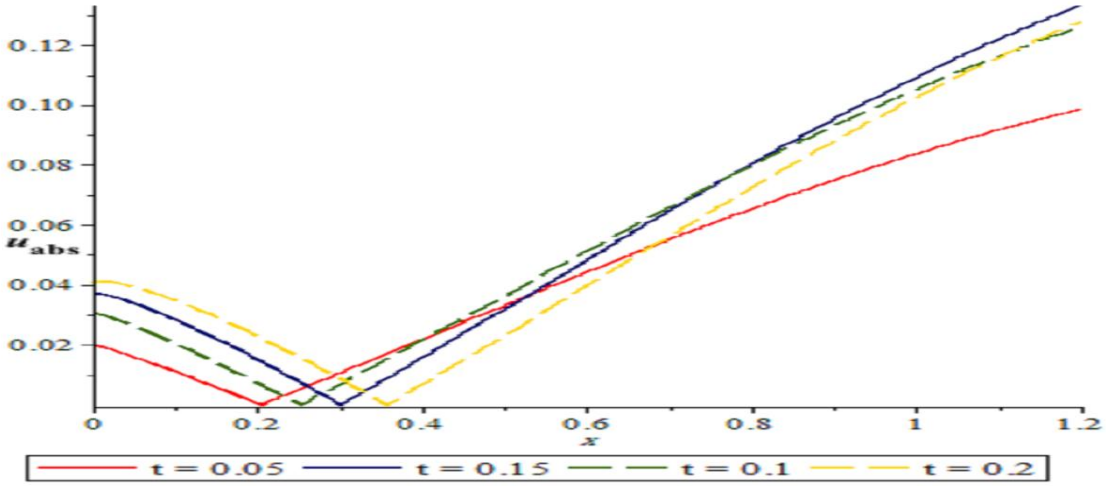



***Figure 4. Plots of the absolute error*** $u_{abs}$ ***when*** $\alpha = 0.7$ ***and*** $\beta = 0.9$ **.**

## Conclusions

In this paper, we have introduced a combination of the homotopy perturbation method and the Sumudu transform method for time-space fractional equations. This combination builds a strong method called the HPSTM. We used the homotopy perturbation Sumudu transform method for solving the Generalized Time-space Fractional Schrödinger Equation with variable coefficients. The HPSTM has been shown to solve effectively, easily and accurately a large class of nonlinear problems with the approximations which convergent are rapidly to exact solutions. The obtained results are compared well with those obtained by VIM, ADM, HAM, and MFVIM. Finally, we conclude that the HPSTM may be considered a nice refinement of existing numerical techniques.

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