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Original article

Oil Well Stimulation Technique (Hydraulic Fracturing Method By PKN, KGD Models of Equation)

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Abstract
Well stimulation has become an essential technique in petroleum engineering to enhance productivity and recovery, particularly in reservoirs characterized by low permeability. Among the commonly applied methods, hydraulic fracturing and acid fracturing are the two primary approaches used to improve hydrocarbon recovery. Hydraulic fracturing, in particular, has proven to be highly effective, as it increases the average permeability in the well's drainage area by creating a conductive channel that allows hydrocarbons to flow more efficiently from the reservoir to the wellbore. This process involves injecting a fluid at a pressure greater than the fracture pressure of the formation, thereby initiating and propagating fractures that extend deeply into the surrounding rock. In this study, the design and evaluation of hydraulic fracturing in the X FIELD sandstone formation in Libya is presented, with a specific focus on applying well-established fracture geometry models—namely the Perkins–Kern–Nordgren (PKN) and the Khristianovic–Geertsma–de Klerk (KGD) models. The research encompasses a comprehensive assessment of reservoir characteristics, rock mechanical properties, and fluid treatment parameters that significantly influence fracture dimensions, including fracture length, width, and height. By integrating these models with field data and conducting a nodal analysis, the study aimed to evaluate the effectiveness of hydraulic fracturing in improving well performance. These outcomes highlight the importance of accurate modeling and proper treatment design in maximizing the benefits of hydraulic fracturing. Overall, the study confirms the critical role of hydraulic fracturing as a stimulation technology for unlocking the potential of tight sandstone reservoirs and recommends its further application across other low-productivity wells in the X FIELD to achieve optimal recovery and improved economic performance. The results obtained from the field application indicated a significant variation in well productivity as follows: For well X1, and before applying the hydraulic fracturing, the productivity index (PI) was calculated at 1.84 STB/day/psi with a baseline production rate (Qo) of 5100 STB/day, the well's conditions improved to 6.8 STB/day/psi and 9000 STB/day after Fracturing, whereas in well X2 the PI increased from 0.34 to 1.4 STB/day/psi and recorded production rate development from 1100 to 2800 STB/day. Finally, one of the further notes of the above findings is that a clear disparity in reservoir productivity between the two wells emphasizes the influence of reservoir variability characteristics and well conditions on production performance.
Keywords: Oil Well, Well Stimulation, Hydraulic Fracturing, PKN Model, KGD Model.

Introduction

Well stimulation is routinely applied to enhance hydrocarbon recovery, especially in low-permeability reservoirs where natural productivity is insufficient to meet economic targets. Techniques such as hydraulic fracturing (commonly known as fracking) have become standard in the oil and gas industry, enabling the creation of highly conductive channels in the formation that extend beyond the near-wellbore region and significantly increase fluid flow. These treatments often involve the use of proppants or acid to maintain fracture conductivity and maximize production [1].

Optimizing hydraulic fracturing requires a careful understanding of reservoir and rock mechanical properties, as well as predictive modeling to design effective treatments and anticipate well performance. Geometry-based models, including the Perkins–Kern–Nordgren (PKN) and Khristianovic–Geertsma–de Klerk (KGD) models, have proven reliable in simulating fracture growth, width, and height under varying conditions, providing guidance for treatment design and post-fracture analysis [2]. This study focuses on the X Field in Libya, characterized by sandstone formations with low permeability and declining production. The research addresses the challenge of insufficient well deliverability by integrating field data with hydraulic fracture modeling, aiming to design stimulation treatments that enhance productivity while ensuring operational feasibility. The outcomes are expected to provide practical insights for improving fracture design and guiding future stimulation practices in similar reservoirs.

Basic concepts of Hydraulic Fracturing

Hydraulic fracturing represents a highly complex process governed by the interplay of fluid mechanics, solid mechanics, fracture propagation, and diffusion phenomena. Each of these factors is interdependent, collectively shaping fracture behavior and influencing the efficiency of stimulation treatments [3]. One of the central aspects of fracture mechanics is fluid leak-off. When fluids—whether Newtonian or power-law—are injected under pressure, they enter fractures and seep into the surrounding pore spaces or initiate secondary cracks. Traditional constant leak-off models often oversimplify this phenomenon, whereas pressure-dependent models, such as those proposed by Carslaw and Jaeger, provide a more realistic representation of fluid behavior in subsurface formations.

The principles of linear elastic fracture mechanics (LEFM) further explain fracture stability and growth. Griffith's energy-based theory establishes that fracture initiation and propagation occur when the energy release rate exceeds the material's

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resistance. The critical stress required for crack opening is determined by parameters such as Young's modulus, surface energy, and crack length. Fracture toughness values—denoted as K_{Ic}, K_{IIc}, and K_{IIIc}—quantify the resistance of a material to crack propagation under different loading modes, serving as essential indicators of rock integrity.

Fracture initiation occurs when fluid is pumped into the formation at a rate exceeding the leak-off capacity. The initiation pressure is influenced by injection fluid pressure, rock type, and surface pump rate. Once this threshold is surpassed, fractures begin to propagate, marking the onset of stimulation. Closely related is the concept of breakdown pressure, which represents the minimum pressure required to overcome in-situ stresses and fracture the rock. Breakdown pressure is affected by rock mechanical properties, fluid type, and the presence of pre-existing fractures. The classical Hubbert and Willis formulation provides a theoretical framework for estimating this critical pressure, offering engineers a valuable tool for designing and predicting fracture initiation.

$$P_b=3S_h-S_v-P_p+T_o \quad \text{...........................Equation 1}$$

Fracture extension in subsurface formations occurs as fractures grow along natural stress planes. The Eaton method provides a framework for accounting for variable overburden stress and the influence of Poisson's ratio, ensuring that fracture propagation is modeled with greater accuracy. The fracturing treatment process is characterized by the dynamic propagation of fractures while fluid simultaneously leaks off into the surrounding rock. Net pressure plays a critical role in keeping fractures open, while the Instantaneous Shut-In Pressure (ISIP) serves as a key indicator for estimating fracture closure. To maintain conductivity, proppant must be continuously supplied; otherwise, sand-out may occur. Interestingly, controlled sand-out can be beneficial if the fracture geometry is sufficient to sustain hydrocarbon flow. Fracture geometry analysis is particularly important in low-permeability reservoirs, where hydraulic fracturing may account for more than 20% of total well costs. Efficient design requires careful consideration of rock mechanical properties, fluid and proppant behavior, and operational constraints. Modern computational models have significantly improved the ability to predict fracture geometry, thereby optimizing treatment strategies and enhancing reservoir productivity.

The role of fracturing fluids is central to hydraulic fracturing operations. These fluids are responsible for opening and extending fractures, creating volume, width, length, and height, while simultaneously transporting proppant along the fracture length. During treatment, fluid flow is typically turbulent in the wellbore and perforations, but transitions to laminar flow within the fractures. Flow effectiveness is influenced by leak-off rate—where high leak-off reduces the fluid available for fracture propagation—and viscosity, which enhances proppant-carrying capacity. Several types of fracturing fluids are employed depending on reservoir conditions. Oil-based fluids are suitable for shallow to medium-depth fractures due to their low injection rates. Water-based fluids, with high density and low friction loss, are preferred for deep wells requiring high injection rates. Emulsions, whether oil-in-water or water-in-oil, offer low fluid loss and adaptability to formation conditions. Foams and gases are particularly effective in very low-permeability formations. Selecting the appropriate fluid requires a thorough understanding of its properties and the ability to adjust them to achieve optimal fracture propagation and proppant transport.

Finally, hydraulic fracturing proppants serve the essential function of keeping fracture walls open after pumping ceases, thereby creating a conductive pathway for hydrocarbons to flow toward the wellbore. Their primary roles are to prevent fracture closure and facilitate reservoir-to-wellbore flow. Ideal proppants must be strong and resistant to crushing and corrosion, possess low density to minimize stress on the formation, exhibit higher permeability than the surrounding rock, and remain cost-effective and readily available. The performance of hydraulic fracturing treatments is strongly influenced by the choice of proppants, which are introduced into fractures to maintain conductivity after pumping ceases. Among the most widely used are natural sand, resin-coated sand, and ceramic beads. Natural sand, or frac sand, is inexpensive and readily available, making it suitable for low-to-moderate pressure environments. Resin-coated sand, with its smoother surface, provides enhanced strength and durability, reducing the risk of crushing. Ceramic beads, though more costly, are exceptionally strong and capable of withstanding high pressures, making them ideal for deep or high-stress reservoirs. Proppants are typically manufactured from high-purity silica (SiO_2) or engineered ceramics, designed to balance strength, density, and permeability.

Equally critical to treatment success is fluid loss control, which ensures that fracturing fluids remain within the fracture to maximize propagation efficiency. Three mechanisms govern fluid loss control. First, viscosity control: increasing fluid viscosity reduces leak-off into the formation. Second, compressibility control: the compressibility of the formation and reservoir fluids limits additional fluid absorption. Third, wall-building control: additives form a temporary filter cake along fracture walls, creating a low-permeability barrier that minimizes fluid loss. The overall effectiveness of fluid loss control depends on the combined action of these mechanisms, and proper management is essential for maximizing fracture extension and treatment efficiency.

The study of fracture models provides insight into fracture geometry and propagation behavior. Hydraulic fracture geometry is primarily controlled by minimum horizontal stress, borehole pressure, and fracturing fluid properties. Fractures propagate when borehole pressure exceeds breakdown pressure and close when subjected to minimum horizontal stress. Classical models such as the Khristianovich–Geertsma–de Klerk (KGD) and Perkins–Kern–Nordgren (PKN) formulations are essentially two-dimensional plane strain approaches, where fluid flow is considered only along the fracture length or radius [4]. Despite their simplifications, these models remain fundamental in predicting fracture geometry and guiding treatment design.

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Perkins and Kern Model of a Vertical Fracture (PKN)

PKN model (Figure 1) is used to simulate the propagation of a vertical hydraulic fracture. It is used for length-to-height ratios of less than 1, this model assumes plane strain in vertical sections and that longer, narrower fractures exist. This model includes the variation in the flow rate along the fracture. The primary assumption of these models is that the fracture length is greater than the height. Assuming that there is no flow i n the vertical direction, the pressure in a vertical cross section of the fracture is constant, and the fracture has an elliptical shape. Also, there is no elastic coupling between the planes, i. e [4]. The state of stress at a point X in the field does not depend on the pressure distribution at other locations along the fracture length. Hence, the fracture width can be expressed as a function of the local pressure. The only coupling between the different vertical cross sections is due to the fluid flow in the fracture [2].

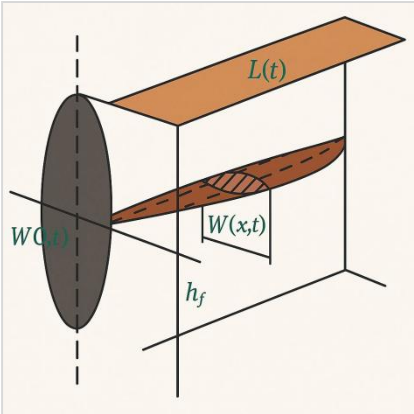

Figure 1 : PKN Fracture model

Perkins and Kern [4] assumed that a fixed-height vertical fracture is propagated in a well-confined pay zone; i.e. the stresses in the layers above and below the pay zone are sufficiently large to prevent fracture growth out of the pay zone. They further assumed that the fracture cross section is elliptical, with the maximum width at a cross section proportional to the net pressure at that point and independent of the width at any other point (vertical plane strain). Although Perkins and Kern developed their solution for non-Newtonian fluids and included turbulent flow, it is assumed here that the fluid flow rate is governed by the basic equation for flow of a Newtonian fluid in an elliptical section. Hydraulic fracture designs for obtaining the fracture width and length were performed by iterating between the Carter technique to obtain (L) , fracture length as a function of time, and the Perkins and Kern model to determine the width, until a consistent solution was found, and then the net fracturing pressure P_{net} is determined [4].

Khristianovich-Geertsma-De Klerk Model (KGD)

The KGD fracture model is one of the fundamental models used to describe the behavior of hydraulic fractures, particularly when the geometry of the fracture is such that the fracture length is smaller than its height. This model becomes most applicable in scenarios where the length-to-height ratio is greater than 3, indicating that the fracture is relatively short and wide in the horizontal direction but extends significantly in the vertical direction [5]. The KGD model assumes a two-dimensional geometry (Figure 2) in which the fracture propagates in a vertical plane and has a rectangular cross-sectional shape. It also presumes plane strain conditions, which means that deformation in the direction perpendicular to the fracture plane is negligible. This assumption simplifies the mathematical analysis of the problem and makes it easier to derive analytical solutions [5].

One of the key features of this model is its treatment of the fracture width distribution. The width of the fracture is calcu lated based on the assumption that the fracture has an elliptical shape in its vertical cross-section, which helps in realistically simulating how the fracture opens in response to the fluid pressure [2]. The model was originally developed by Khristianovich, Zheltov, Geertsma, and de Klerk, who derived equations that describe the width of vertical fractures created by fluid injection. These equations take into account the fluid flow within the fracture and its interaction with the surrounding rock formations [5]. In the KGD model, the fluid is injected into a fracture with a finite length but infinite height, and it flows from the wellbore along the length of the fracture. The pressure distribution and flow rate are governed by fluid mechanics and rock elasticity. The model is especially useful for analyzing hydraulic fracturing in thick formations, where the vertical growth of the fracture dominates over the horizontal expansion [2]. Furthermore, the KGD model assumes laminar flow within the fracture, and it is often applied in cases where the surrounding rock has high vertical stress and the fracture tends to grow vertically rather than horizontally [5].

The design process for determining the fracture width and length involves an iterative approach. The Carter leak-off model is used to estimate fracture length as a function of time. Simultaneously, the KGD model is applied to estimate the fracture width, until a consistent solution is reached. Once consistency is achieved, the net fracturing pressure is calculated [5].

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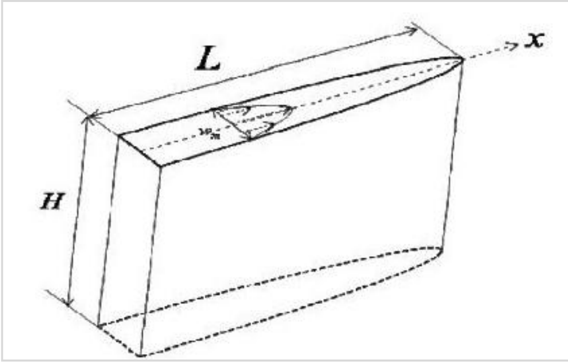

Figure 2 : illustrates the KGD fracture geometry

In the context of hydraulic fracturing, it is commonly observed that the fracture propagates more extensively in the horizontal direction than it does vertically. This is due to the differences in stress distribution and rock mechanical properties, which generally restrict the vertical growth of the fracture. As a result, the fracture tends to expand laterally along the fracture plane more significantly than it does upward or downward. Therefore, it can be concluded that vertical fracture growth is typically less pronounced compared to horizontal growth [5].
From a modeling perspective, this behavior is influenced by the assumptions made in different fracture propagation models. For instance, the PKN (Perkins-Kern-Nordgren) model is based on the assumption that the fracture height remains constant and is much smaller than the fracture length. In this model, each vertical cross-section of the fracture is considered to behave independently. This implies that the fluid pressure distribution is primarily governed by the height of the fracture section rather than its length [5]. Such an assumption is only valid when the fracture is relatively long and its height is limited, making the vertical deformation negligible in comparison to horizontal expansion. Conversely, other models might assume a constant length and allow the height to vary, which would result in different stress and pressure profiles and ultimately affect the predicted geometry of the fracture. The fundamental difference between these modeling approaches lies in which dimension (height or length) is considered dominant or constrained. This key distinction leads to divergent methodologies for solving the hydraulic fracturing problem and results in different predictions for the final shape and size of the fracture. Understanding these assumptions is crucial when selecting the appropriate model for a specific geological setting or engineering objective.

Case Study
Introduction to the X-Field
X-Field is located in Concession 59 with its northern flanks extending to Concession D. It was discovered with the drilling and testing of the X1 well in early 2002; the well intermittently produced for a total of 129 days with cumulative production of 338 MBO, representing an average rate of 2620 BOPD. Appraisal wells were subsequently drilled to define the limits of the field, fluid contacts, and the characteristics of the reservoirs [6].

X-Field structural overview
Structurally, the field is bisected by numerous faults, the western side of the field is dominated by NNW-SSE-trending faults, while the eastern side is dominated by faults trending WNW-ESE. The Nubian deposition is believed to have been generally uniform in thickness, then modified by the structural activity (uplift, faulting, and erosional truncation) that was responsible for creating the current field geometry. Also shows the stratigraphic compartmentalization that has resulted from the sub-cropping units [6].
Figure 3 illustrates the structural and stratigraphic elements that characterize the X field trap. The section trends NE-SW and demonstrates the down dip preservation of both upper and lower Nubian and updip erosion and loss of the upper Nubian. Most fault offsets have sand-on-sand juxtaposition due to the thickness of the Nubian. However, along the major NNW-SSE fault (at the 4C-1 well above) the Nubian is juxtaposedagainst the Lower Sirte shale and is expected to seal or partially seal. The top seal is the overlying Upper Cretaceous marine shales of the Rakb Group, referred to as the Lower and Upper Sirte Shales. The lateral seal for the reservoir sands is interpreted to be formed by impermeable, tight Nubian sections and/or impermeable granite basement [6].

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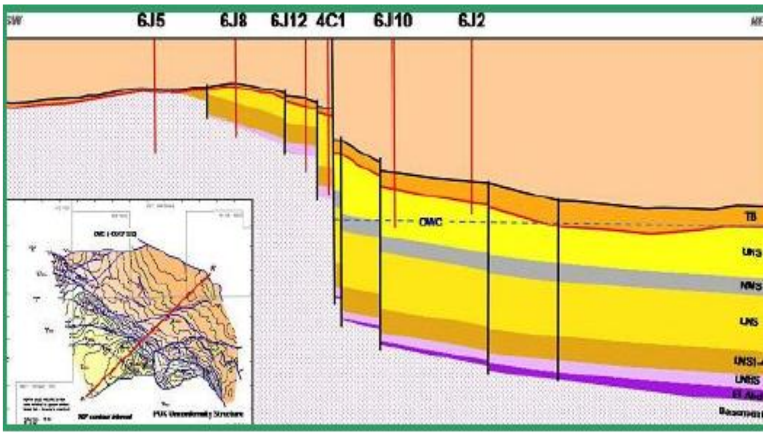

Figure 3 : Schematic cross-section & the proposed structural/stratigraphic model

The selected oil well information

a) Well X1

Well X1 was drilled in X-field to 12,950 feet and completed in UNSS; the well history is shown below [7] :

Table 1: X1 History

Year	Operation	Q(BOPD)	WHP(PSI)	NOTE
2004	Initial production after drilling	3984	990	Well was shut in shortly after
Dec 2004	Decline observed	850	160	Significant drop in production
2008	Uses alcoholic acid	2155	-	Improved rate, but did not reach initial production
2013	Massive hydraulic fracturing by Halliburton	7000	1075	Post-frac test by Schlumberger showed major improvement

b) Well X2

Well X2 was drilled in X-field to 13,005 feet and completed in LNSS; the well history is shown below:

Table 2: X2 History

Year	Operation	Q(BOPD)	WHP(PSI)	NOTE
Jul 2013	Drilled and evaluated	-	-	Porosity: 7.9%• Permeability: 3 md
Dec 2013 – Jan 2014	Pre-frac test	385	172	Low production due to poor reservoir quality
Jan 2014	Hydraulic fracturing of LNSS	3903	580	Significant improvement post-fracturing

The Methodology of the process

Data collection and analysis

The study commenced with the systematic collection and analysis of reservoir and well data for two representative wells, X1 and X2. The dataset comprised bottom hole pressure, reservoir pressure, fluid properties, production history, and completion details [8]. This comprehensive dataset served as the foundation for all subsequent modeling and analytical procedures, ensuring that the fracture simulations were firmly grounded in reservoir realities. Building upon this dataset, two widely recognized fracture modeling approaches were employed: the Perkins–Kern–Nordgren (PKN) model and the Khristianovic–Geertsma–de Klerk (KGD) model. These formulations were solved to estimate fracture width, length, and propagation behavior under varying reservoir and operational conditions. Their application provided critical insights into fracture geometry and the dynamic response of the reservoir to hydraulic stimulation.

To further evaluate operational strategies, multiple treatment scenarios were defined by varying injection rate and injection volume [2]. These hypothetical cases were carefully selected to explore the influence of different injection strategies, thereby identifying the most effective operational window. The objective was to maximize production while maintaining safe and efficient fracturing behavior, balancing reservoir mechanics with operational constraints. Finally, production performance analysis was conducted using a combined approach of Inflow Performance Relationship (IPR) and Nodal Analysis. The IPR curve, representing reservoir inflow, was plotted against the VLP/System curve, which characterizes wellbore and surface outflow. The intersection of these curves provided a quantitative basis for determining optimum production rates before and after fracturing, as well as the bottom hole flowing pressure. This analysis enabled comparative

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productivity assessments across scenarios and highlighted the impact of fracture conductivity improvements on overall well deliverability.

Data Required for Hydraulic Fracturing Design.

Input data are shown in

Table 3 is then used to design and optimize fracture treatment. During the optimization process, sensitivity studies on placement of perforations, treatment size, and fluid and proppant volumes are performed [7], [8].

Table 3: Data Required

Data	WellX1	Well X2	Units
ProducingInterval	12380-12754	12624-12730	Ft
Formation Thickness	286	286	Ft
Depth	12950	13005	Ft
AverageReservoir Pressure	6094	6000	Psi
Horizontal TensileStrength of Rock	825	225	Psi
OilCompressibility	0.0000103	0.000014	psi^-1
Water Compressibility	0.000099	0.000154	psi^-1
Gas Compressibility	0.0000034	0.0000042	psi^-1
Oil Saturation	0.79	0.79	%
Water Saturation	0.19	0.19	%
Gas Saturation	0.02	0.02	%
Formation Porosity	7%	8%	%
Formation Permeability	4.8	5.5	Md
MatriXfieldCompress. Transient Time (Δtma)	52	52	
MatriShearTransientTime (Δtsma)	82	82	
FracturingFluidViscosity	99	99	Cp
FracturingFluid Density	8.36	8.36	Ppg
ReservoirOilViscosity	0.5	2.903	Cp
Area of Filter Medium	7.84	7.84	Cm^2
Slope of FluidLossCurve at Lab.:	0.01	0.01	Gal.ft^2
Filtration Pressure at Lab	100	100	Psi
Casing Outer Diameter	9	7	In
WellboreDiameter	8.5	7	In
Drainage Area	2040.004	2139.6172	In^2
Proppant Size and Type	20/40	20/40	Mesh
Porosity of PackedProppan	4%	4%	%
Specific Gravity of Proppant	3.5	3.6	
FracturingFluidspurtloss	0.01	0.01	Cm/min^0.5
Tubing InnerDiameter	3	2.75	In
Tubing Depth	12281	124111	Ft
Gas Oil Ratio	988	1260	Scf/bbl
Bubble Point Pressure	4350	4350	Psi
Bottom Hole Temperature	290	305	F
Perforation Diameter	0.4	0.4	In
PerforationDischarge Coefficient	0.75	0.72	
Number of Perforations	5	5	Shot/ft
Frictional Pressure Gradient	0.69	0.69	Psi/ft

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Oil Formation Volume Factor	1.32	1.35	b/stb

Hydraulic fracturing design procedures and results:
This section presents the main data required for hydraulic fracture design and the design procedure steps [3] ; (i. e., step by step).

a) Calculating the overburden pressure as follows:
$P_{ob}=G_{ob}*D$ ....Equation 2
P_{ob} for Well X1 = 32504.5 psi
P_{ob} for Well X2= 33292.8psi

b) Calculating the the minimum fracturing pressure (Pf)
$P_F = \frac{2*\left(\frac{V}{1-V}\right)(\rho_{Ob}-P_P)+T_O}{2-\alpha\left(\frac{1-2V}{1-V}\right)+P_P}$ ....Equation 3
P_f for Well X1=11392.4827 psi
P_f for Well X2=1023.2234 psi

c) Calculation of fracturing fluid coefficients
$c_v = 0.0469\,[\frac{k\Delta p\phi}{\mu_{frac}}]^{1/2}$ ....Equation 4
C_V for Well X1 =0.210334ft/min^0.5
C_V for Well X2=0.23055988ft/min^0.5
$c_c = 0.0374\Delta p[\frac{kc_f\phi}{\mu_{res}}]^{\frac{1}{2}}$ ......Equation 5
C_C for Well X1 =0.48142562ft/min^0.5
C_C for Well X2= 0.24530039 ft/min^0.5
$c_w = \frac{0.0164m_{act}}{A_f}$ ....Equation 6
C_W for Well X1 = 0.00015227 ft/min^0.5
C_W for Well X2= 0.00014678 ft/min^0.5

d) Calculation the effective fracturing fluid Coefficient:
ΔP (closure stress) = Pf – PP ....Equation 7
ΔP (closure stress) for Well X1 = 5298.4827 PSI
ΔP (closure stress) for Well X2= 4923.22343 PSI

e) Read KFK_FKF directly from the chart at the intersection of closure Pressure and the appropriate Mesh Size line.
For Well X1 = 100000 md
For Well X2=100100 md
The Fracture Volume (Fracture Dimensions):
Assume (qi = bbl/min), and (Vi = bbls):

f) Calculating pimping time (tp):
$t_P = \frac{V_I}{G_I}$ ....Equation 8

g) Calculating fracture volume (Vf)
We assume 7 cases to find the best case with PI and calculate fracture volume (Vf):
$V_F = \frac{\pi}{2} * L * W_W * h_F$ ....Equation 9

h) Calculating the fracture efficiency (Eff):
Eff = (Vf/Vi)*100% ....Equation 10

Table 4: Results of theoretical scenarios applied to well X1

Cases	QI (bbl)	Vi (bbl/min)	Wf (ft)	Lf (ft)	Vf(ft^3)	Efficiency	PI
1-1	3750	10	0.25	710	6641.754	31.54 %	6.8
1-2	4000	10	0.25	750	7015.94	31.23 %	7.2
1-3	3500	10	0.25	710	6641.75	33.79 %	6.8
1-4	4500	10	0.27	800	7981.3	31.99 %	7.5
1-5	6000	10	0.3	1000	11225.5	33.32 %	8.3
1-6	5500	10	0.29	925	10037	32.51 %	7.9
1-7	5000	10	0.26	850	8269.45	29.45 %	7.5

Table 5: Results of theoretical scenarios applied to well X2

Cases	QI (bbl)	Vi (bbl/min)	Wf (ft)	Lf (ft)	Vf	Efficiency	PI
1-1	6500	10	0.32	1000	11973.7	32.8%	1.4

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1-2	1500	10	0.2	390	2993.47	34.6%	1.01
1-3	2500	10	0.24	525	4714.71	33.5%	1.2
1-4	5000	10	0.3	810	9092.7	32.2%	1.4
1-5	4000	10	0.28	700	7333.99	32.6%	1.3
1-6	4500	10	0.29	790	8572.5	33.9%	1.4
1-7	6000	10	0.31	920	10671.7	31.6%	1.3

Fracture half-length.vs. fracture width (using KGD And PKN) For 7 cases are shown in Figure 4 & Figure 5.

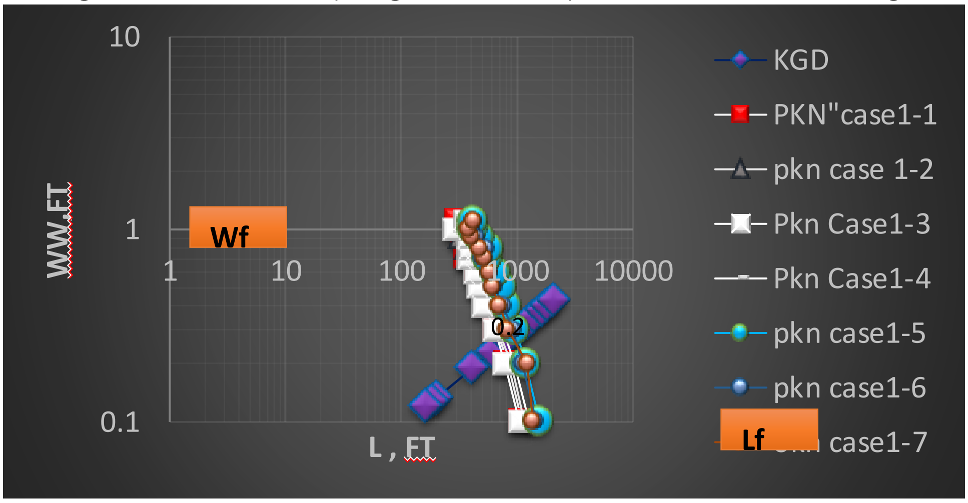

Figure 4 : Ww V.s L for Well X1

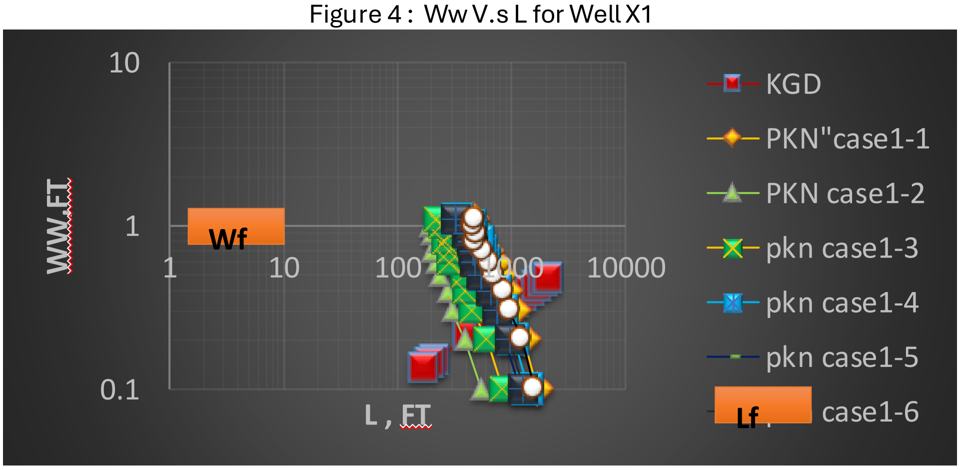

Figure 5 : Ww VS L For Well X2

i) Analyzing the Well's production performance using IPR Curves
Establishing IPR curves could be done as follows:
Firstly, calculating the productivity Index (PI=Jo) **b**efore fracturing achieved can be done using Darcy's law as follows:

$$J_O = \frac{q_o}{(P_e - P_{wf})} = \frac{7.08*10^{-3}*K*h}{\mu_o*\beta_o*\ln\frac{re}{rw}} \text{...................Equation 11}$$

The results here below:
(For Well X1) → PI = Jo = 1.84 B/D /psi
(For Well X2) → PI = Jo = 0.34 B /D/psi

Second, calculating the productivity Index (PI=Jo) after fracturing achieved can be done as follows:
calculate **L / R_e** and **relative permeability (k_r),** then read **Jf / Jo** from the chart; the values as follows:
(For Well X1) → Jf/Jo = 3.7
(For Well X2) → Jf/Jo = 4.1
Then by using Jf , we could calculate PI or Jo, and the results were as follows:
The productivity index after fracture is
(For Well X1) → PI = Jo = 6.8 B/D/psi
(For Well X2) → PI = Jo = 1.4 B/D/psi

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Thirdly, we plotted the IPR curve (before & after frac) for the two wells (Figure 6 & Figure 7). From the plots, it was clear that the bubble point pressure is below the reservoir pressure, indicating a composite IPR type. These plots are used to show the IPR relationship and nodal analysis for the wells.

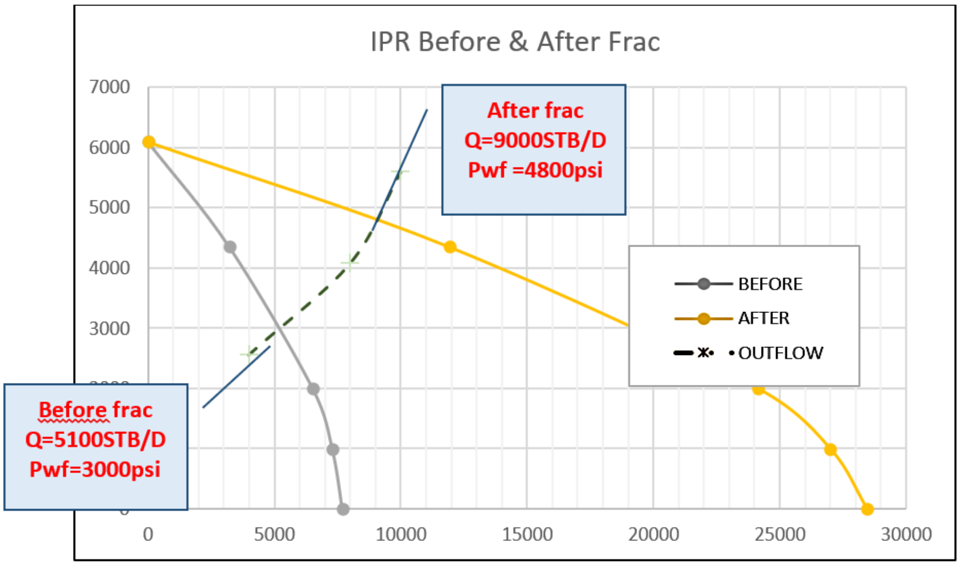

Figure 6 : The IPR plot for Well X1

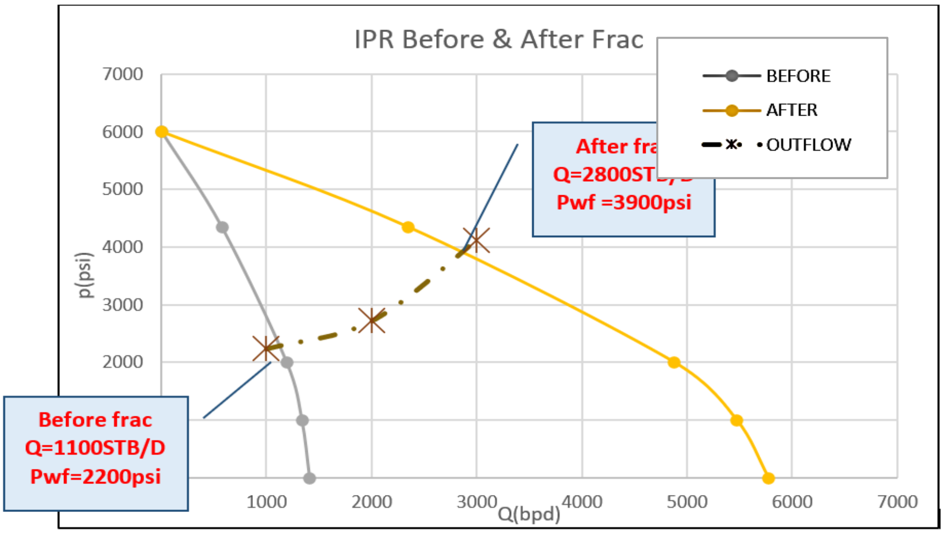

Figure 7 . The IPR plot for Well X2

Conclusions

Evaluating the hydraulic fracturing for this study has indicated that the production rate (Q) was increased, that verified by comparing the flow rate and bottom flowing pressure (Pwf) after fracturing with those before fracturing; it is as follows:

a) For Well X1 :
   Optimum Q (before fracturing) = 5100 bbl/day. it greatly increased to 9000 bbl/day after hydraulic fracturing was done, Optimum Pwf, before fracturing, was 3000 psi, it raised to 4800 psi after fracturing.
   The increase in productivity is = 3.69 times, much better.
b) For Well X2:
c) Q_o optimum (before fracturing) = 1100 bbl/day. After the hydraulic fracturing treatment was applied, the producing rate was greatly increased to a value of: Q_o optimum (after fracturing) = 2800 bbl/day
d) Pwf optimum (before fracturing) = 2200 psi . than became Pwf optimum (after fracturing) = 3900 psi .
e) The increase in productivity is = 4.11 times, so much better.

Recommendations.

Our recommendation is to apply the same techniques for other wells where the production recorded severe decline.

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