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Original article

Component-wise Validation of Amplitude Encoding within Quantum Amplitude Estimation

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Abstract
Quantum Amplitude Estimation (QAE) is a fundamental component of many quantum computing algorithms and applications. The practical implementation of QAE workflows on current Noisy Intermediate-Scale Quantum (NISQ) platforms requires accurate probability amplitude representation and reliable measurement recovery. Understanding the behavior of individual computational stages is therefore essential for improving the overall performance of practical quantum estimation procedures. This study presents a component-wise validation of the amplitude encoding process within a controlled quantum simulation environment. The experimental implementation was developed using the Qiskit framework, where classical probability values were encoded into single-qubit quantum states through parameterized rotation operations and subsequently recovered from measurement statistics. A systematic evaluation was performed across the probability interval from 0.1 to 0.9 using 8192 measurement shots for each experiment. The experimental results demonstrate that the recovered measurement probabilities closely match the corresponding target values throughout the investigated domain. Statistical evaluation produced a Mean Absolute Error (MAE) of 0.003521, a Mean Squared Error (MSE) of 1.67×10^{-5}, and a Root Mean Squared Error (RMSE) of 0.004087, indicating high numerical consistency under ideal simulation conditions. The findings suggest that the amplitude encoding stage accurately represents classical probability values and provides a reliable baseline for studying more advanced quantum estimation workflows. The component-wise validation methodology adopted in this work offers a practical framework for investigating individual stages of quantum algorithms and supports future research on hybrid quantum-classical processing strategies for NISQ-era quantum computing.
Keywords: Quantum Amplitude Estimation, Amplitude Encoding, Quantum Simulation, Qiskit.

Introduction
Quantum Amplitude Estimation (QAE) is one of the fundamental algorithmic primitives in quantum computing, offering a quadratic improvement in sampling efficiency over classical Monte Carlo methods. Since its introduction, QAE has become an essential component of numerous quantum algorithms and applications, including quantum finance, risk analysis, quantum machine learning, optimization, and the simulation of complex physical systems [1].
The efficient estimation of probability amplitudes makes QAE an important building block for achieving practical quantum advantage in computational tasks. Despite its theoretical significance, the practical implementation of QAE on current Noisy Intermediate-Scale Quantum (NISQ) hardware remains challenging. Existing quantum devices are affected by several physical limitations, including gate imperfections, decoherence, measuring noise, and restricted circuit depth. These limitations introduce deviations between theoretically expected amplitudes and experimentally observed outcomes, reducing the reliability of practical amplitude-estimation workflows. Traditional research has primarily focused on improving quantum circuit architectures or mitigating hardware-induced noise through error-correction and error-mitigation techniques; however, practical amplitude-estimation workflows consist of multiple interconnected stages, including state preparation, amplitude encoding, quantum evolution, measurement, and classical post-processing. The contribution of each stage to the overall estimation error is not always explicitly characterized, making it difficult to identify the dominant sources of systematic deviations.
A component-wise validation methodology is adopted in the present study to investigate the behavior of practical amplitude-estimation workflows. Rather than treating the estimation process as a single computational block, its principal components are examined individually. Experimental evaluation within the Qiskit simulation framework indicates that the amplitude encoding stage accurately reproduces target probability amplitudes across the investigated domain, exhibiting only minor reconstruction errors under controlled simulation conditions. These observations suggest that substantial estimation deviations are more likely to arise during subsequent estimation and recovery procedures than during the state preparation stage itself. Motivated by these observations, the Branch-Aware Probability Inversion (BAPI) framework is introduced as a branch-aware classical post-processing strategy for investigating and mitigating systematic deviations in amplitude-estimation workflows. The proposed framework is based on a geometric interpretation of probability evolution and incorporates branch identification together with polynomial calibration to improve the consistency of recovered amplitudes. Rather than replacing existing quantum estimation procedures, the framework complements them through a hybrid quantum-classical processing strategy compatible with NISQ-era computational constraints. [2]

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The principal contributions of this study are summarized as follows:
A component-wise validation methodology for practical amplitude-estimation workflows is presented. The accuracy of the amplitude encoding stage is experimentally evaluated using quantum simulation. Systematic deviations arising during practical estimation and recovery procedures are investigated. The Branch-Aware Probability Inversion (BAPI) framework is proposed as a branch-aware post-processing strategy for mitigating estimation deviations. Experimental evaluation is conducted within the Qiskit simulation environment to assess the practical behavior of the proposed methodology and its implications for hybrid quantum-classical processing.
The remainder of this paper is organized as follows. Section 2 presents the theoretical background of Quantum Amplitude Estimation and the practical challenges associated with NISQ implementations. Section 3 describes the proposed Branch-Aware Probability Inversion methodology. Section 4 outlines the experimental framework and validation procedures. Section 5 presents experimental results and analysis. Section 6 discusses the implications and limitations of the proposed approach, and Section 7 concludes the study while highlighting potential directions for future research.

Theoretical background
Quantum Amplitude Estimation (QAE) is one of the fundamental algorithmic primitives in quantum computing. It provides an efficient mechanism for estimating probabilities associated with quantum states and offers a quadratic improvement in sampling complexity compared with classical Monte Carlo methods. This computational advantage has established QAE as an important component in applications such as quantum finance, optimization, machine learning, and the simulation of physical systems. [3] [4]

Quantum amplitude estimation
The objective of Quantum Amplitude Estimation is to determine the probability associated with a desired quantum state.
A single-qubit quantum state may be represented as:

$$\Psi = \sqrt{(1 - a)}\, |0\rangle + \sqrt{a}\, |1\rangle \qquad (1)$$

Were $0 \le a \le 1$, and a denotes the probability associated with the target state $|1\rangle$.
The amplitude encoding process transforms a classical probability value into a quantum state through a rotation operation. The corresponding rotation angle is defined by:

$$\theta = 2 \arcsin(\sqrt{a}) \qquad (2)$$

Applying the rotation gate $Ry(\theta)$ to the initial state $|0\rangle$ produces:

$$Ry(\theta)|0\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)|1\rangle \qquad (3)$$

The probability of measuring the target state is

$$P(1) = \sin^2(\theta/2) \qquad (4)$$

Since $\sin^2(\theta/2) = a$
The measured probability reproduces the original classical value.
This relationship establishes the mathematical connection between classical probability values and their corresponding quantum representations.

Practical Challenges in NISQ Systems Although the theoretical
properties of Quantum Amplitude Estimation are well established, practical implementations on Noisy Intermediate-Scale Quantum (NISQ) systems introduce several challenges.
Current quantum hardware is affected by various physical limitations, including:
• Gate imperfections, Decoherence, Measurement noise, Limited qubit connectivity, Restricted circuit depth. [5]
These limitations may introduce deviations between theoretical expectations and experimentally observed results.
Practical amplitude-estimation workflows also involve multiple computational stages, including:
• State preparation, Amplitude encoding, Quantum evolution, Measurement, Classical post-processing.
The interaction among these stages can influence the overall estimation accuracy and complicate the identification of dominant error sources.

Systematic Deviations in Practical Estimation
In practical implementations, recovered probability estimates may deviate from their target values.

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Unlike purely statistical fluctuations, systematic deviations exhibit structured behavior across the probability domain and may persist under repeated measurements. Small inaccuracies introduced during intermediate computational stages can propagate through non-linear transformations, resulting in larger deviations in the final recovered estimates.
A practical strategy for investigating these effects is to validate the individual components of the estimation workflow independently. By separating state preparation from subsequent estimation and recovery procedures, the contribution of each computational stage can be examined under controlled conditions. [6]
This component-wise perspective provides a clearer understanding of practical estimation behavior and establishes a foundation for subsequent post-processing methodologies.

Hybrid Quantum-Classical Processing

Hybrid quantum-classical computation has emerged as an effective approach for addressing the limitations of current NISQ technology.
In hybrid computational workflows, quantum processors perform state preparation and measurement, while classical algorithms refine, calibrate, or interpret the measured results. Such post-processing strategies may improve estimation consistency without increasing quantum circuit complexity. [7]
The present study adopts this hybrid perspective by investigating branch-aware post-processing techniques designed to improve the recovery of probability estimates. The theoretical concepts presented in this section provide the mathematical and conceptual foundation for the Branch-Aware Probability Inversion (BAPI) framework introduced in the following section.

Branch-Aware Probability Inversion (BAPI) Methodology

the Branch-Aware Probability Inversion (BAPI) framework. The proposed methodology is formulated as a hybrid quantum-classical post-processing strategy for investigating the recovery of probability amplitudes in practical quantum estimation workflows. [8]

Motivation

Practical quantum estimation workflows involve several computational stages, including state preparation, amplitude encoding, quantum evolution, measurement, and classical recovery. Although ideal amplitude encoding accurately maps classical probabilities into quantum states, subsequent estimation procedures may involve non-linear transformations that complicate the interpretation of measured probabilities [9]. Instead of treating the measured probability as a direct estimate of the target value, the BAPI framework considers the possibility that the recovered measurement may correspond to different recovery branches generated during the estimation process. The objective is to investigate whether branch-aware classical processing can provide a structured interpretation of recovered probability values [10].

3.2 Branch-Aware Inversion

Let P denote the recovered probability obtained from a quantum estimation procedure.
The branch-aware inverse mapping is defined as:

$$\hat{v} = \sin^2(\arcsin(\sqrt{P})/(2k + 1)) \quad (5)$$

where $\hat{v}$ denotes the recovered amplitude estimate, P represents the measured probability, k denotes the branch index. The factor $(2k + 1)$
accounts for the rotational structure associated with repeated quantum transformations.
For the simplest case, $k = 0$, Equation (5) becomes

$$\hat{v} = P \quad (6)$$

which corresponds to direct probability recovery.
Alternative branch indices generate different inverse mappings that may be explored during the classical post-processing stage. The BAPI framework does not modify the underlying quantum circuit but instead provides an additional interpretation layer for recovered probability values.

3.3 Polynomial Calibration

Although the branch-aware inversion provides an initial recovered estimate, residual deviations may remain because of imperfections in practical estimation workflows.
A polynomial calibration function is introduced to refine the recovered amplitude.

$$f(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0 \quad (7)$$

where a_3, a_2, a_1, and a_0 are calibration coefficients determined from reference data. The corrected estimate is obtained as

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$$v_c = f(\hat{v}) \quad (8)$$

where vc denotes the calibrated recovered amplitude.
This calibration stage operates entirely in the classical domain and does not increase the complexity of the underlying quantum circuit.

Hybrid Processing Workflow

The BAPI framework consists of four sequential stages.
Step 1 Encode the target probability amplitude into a quantum state.
Step 2 Execute the quantum estimation procedure and obtain the recovered probability P.
Step 3 Apply the branch-aware inverse transformation to compute the preliminary estimate $\hat{v}$.
Step 4 Apply polynomial calibration to obtain the corrected estimate v_c.
This workflow separates quantum computation from classical refinement and provides a hybrid processing strategy compatible with current NISQ computational limitations.

Conceptual Advantages

The proposed BAPI framework provides several practical advantages.
- The original quantum estimation procedure remains unchanged.
- The refinement process is performed entirely through classical post-processing.
- No additional quantum gates or qubits are required.
- The methodology is compatible with existing amplitude-estimation workflows.
- The framework provides a structured mechanism for investigating systematic estimation behavior.

Rather than replacing existing quantum estimation techniques, BAPI serves as a complementary post-processing methodology that may facilitate the analysis and interpretation of practical estimation workflows.

Experimental Setup

The experimental investigation was designed to examine the practical behavior of probability amplitude recovery within a controlled quantum simulation environment. Particular attention was given to the accuracy of the amplitude encoding process and the subsequent recovery of measurement probabilities, providing a baseline for evaluating branch-aware post-processing strategies.

Computational Environment

The experimental platform was implemented using the Qiskit quantum computing framework. This environment provides a flexible infrastructure for constructing quantum circuits, executing simulations, and collecting measurement statistics under reproducible conditions.
The study focused on single-qubit circuits, allowing the relationship between classical probabilities and their quantum representations to be investigated without additional complexity introduced by multi-qubit interactions.
The computational workflow consisted of the following stages:
- Preparation of the initial quantum state.
- Amplitude encoding through a parameterized rotation gate.
- Measurement of the quantum state.
- Recovery of probability values from measurement statistics.
- Classical evaluation of estimation accuracy.

This structured workflow enables individual computational stages to be analyzed separately while maintaining a consistent experimental methodology.

Amplitude Encoding Procedure

The experimental implementation begins by converting a classical probability value into a quantum state.
For a target probability v, the corresponding rotation angle is determined according to Equation (2):

$$\theta = 2 \arcsin(\sqrt{v}) \text{ A} \quad (9)$$

single Ry rotation is then applied to the initial state |0⟩, generating the encoded quantum state. Following repeated circuit execution, the probability of observing the state |1⟩ is estimated from the measurement outcomes.

$$P = N_1 / N \quad (10)$$

where N_1 represents the number of measurements yielding the state |1⟩, and N denotes the total number of circuit executions.
The recovered probability is compared with the original target value to evaluate the fidelity of the encoding process.

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Experimental Parameters

A uniform experimental configuration was maintained throughout the investigation to ensure consistency across all test cases.

Table 1. Experimental Configuration

Parameter	Configuration
Quantum Framework	Qiskit
Circuit Architecture	Single-Qubit
Encoding Operation	Ry Rotation
Measurement Basis	Computational Basis
Number of Shots	8192
Target Probability Range	0.1 – 0.9
Simulation Mode	Ideal Quantum Simulation

The selected probability interval provides representative coverage of practical operating conditions while avoiding trivial extreme cases.

Validation Strategy

A stage-by-stage validation approach was adopted to investigate the behaviour of the estimation workflow. Rather than evaluating the complete process as a single computational unit, individual components were examined independently.

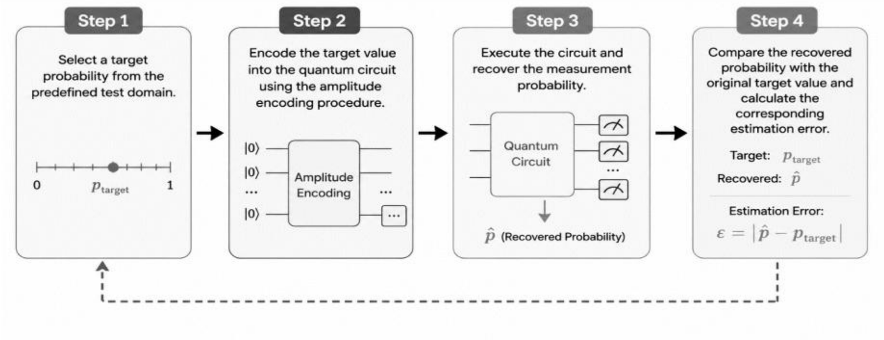

Figure 1. Validation Strategy

This methodology provides a systematic mechanism for identifying the contribution of each computational stage to the overall estimation performance.

Statistical Evaluation

The quality of the recovered probability amplitudes was assessed using standard statistical measures.
The Mean Absolute Error (MAE) is defined as

$$\text{MAE} = (1/n)\, \Sigma\, |vi - vi\_est| \qquad (11)$$

where vi is the target probability, and vi_est is the recovered estimate. The Mean Squared Error (MSE) is given by

$$\text{MSE} = (1/n)\, \Sigma\, (vi - vi\_est)^2 \qquad (12)$$

The Root Mean Squared Error (RMSE) is calculated as

$$\text{RMSE} = \sqrt{\text{MSE}} \qquad (13)$$

These metrics provide complementary information regarding estimation accuracy and the overall stability of the recovery process.

Experimental Evaluation Protocol

Experimental evaluation was conducted across the target probability interval $0.1 \le v \le 0.9$. For each target value, an independent quantum circuit was generated and executed using 8192 measurement shots.
Measurement outcomes were converted into probability estimates according to Equation (10), and the recovered values were compared with their corresponding targets.
The resulting dataset was analyzed using the statistical measures introduced in the previous subsection. This evaluation protocol establishes a reproducible baseline for assessing the behavior of practical amplitude recovery and provides the reference framework for the experimental results presented in the following section.

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Results

This section presents the experimental results obtained from the Qiskit simulation framework. The primary objective was to evaluate the accuracy of the amplitude encoding procedure and establish a baseline for future hybrid quantum-classical processing strategies.

Amplitude Encoding Performance

Target probabilities in the range $0.1 \le v \le 0.9$ were encoded into single-qubit quantum states and executed using 8192 measurement shots. The recovered probabilities closely matched the target values across the domain investigated.

Table 2. Amplitude Encoding Results

Target	Measured	Absolute Error
0.1	0.0956	0.0044
0.2	0.2080	0.0080
0.3	0.2958	0.0042
0.4	0.4047	0.0047
0.5	0.4984	0.0016
0.6	0.5977	0.0023
0.7	0.7011	0.0011
0.8	0.8014	0.0014
0.9	0.8960	0.0040

The recovered probabilities exhibit only minor deviations from the corresponding target values.

Statistical Performance

The overall accuracy of the encoding process was evaluated using standard statistical measures.

Table 3. Statistical Performance Metrics

Metric	Value
MAE	0.003521
MSE	0.0000167
RMSE	0.004087

The low error values indicate that the amplitude encoding procedure achieves high numerical accuracy under ideal simulation conditions.

Discussion

The experimental results demonstrate that classical probability values can be accurately encoded into single-qubit quantum states and reliably recovered through quantum measurement. The small reconstruction errors suggest that the encoding stage itself is not a significant source of estimation inaccuracies within the investigated domain.
These findings support the component-wise validation methodology adopted in this study and provide a reliable baseline for future investigations of hybrid quantum-classical post-processing techniques.

Summary

The principal findings of the experimental investigation are:
Amplitude accurately reproduces target probability values.
- Recovered measurement probabilities closely match the expected results.
- Statistical error measures remain consistently low across the range investigated.
- The obtained baseline provides a practical reference for future studies of quantum estimation workflows.

The experimental results demonstrate that the amplitude encoding procedure accurately reproduces target probability values under ideal quantum simulation conditions. The low statistical error metrics indicate that the encoding stage itself introduces only minor deviations across the investigated probability domain.
These findings support the component-wise validation methodology adopted in this study. By analyzing individual stages of the quantum estimation workflow separately, the contribution of each computational component can be evaluated more effectively. In particular, the results suggest that substantial estimation inaccuracies observed in practical quantum algorithms are likely associated with later computational stages rather than with the amplitude encoding process itself.
The study also highlights the potential of hybrid quantum-classical processing strategies. Reliable quantum state preparation combined with classical analysis provides a practical framework for improving the interpretation of measurement outcomes without increasing quantum circuit complexity.

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Several limitations should be acknowledged. The experiments were conducted under ideal simulation conditions and focused on single-qubit amplitude encoding. Practical quantum hardware introduces additional sources of error, including noise, decoherence, and gate imperfections, which were beyond the scope of the present investigation. Furthermore, extending the analysis to multi-qubit systems and more advanced estimation procedures remains an important direction for future research.

Overall, the findings establish a reliable experimental baseline for studying practical quantum estimation workflows and support further investigation of hybrid quantum-classical methodologies for quantum computing applications.

Conclusion

This study investigated the practical behavior of probability amplitude encoding within a controlled quantum simulation environment. A component-wise validation methodology was adopted to examine the accuracy of individual stages of the quantum estimation workflow and to establish a reliable experimental baseline.

The experimental results demonstrated that the amplitude encoding procedure accurately reproduces target probability values across the investigated domain. The recovered measurement probabilities showed close agreement with the corresponding target amplitudes, while the statistical performance metrics indicated a high level of numerical consistency under ideal simulation conditions.

The study also explored the role of hybrid quantum-classical processing from a conceptual perspective. By separating quantum computation from classical analysis, component-wise validation provides a systematic framework for investigating the behavior of practical quantum estimation workflows without increasing quantum circuit complexity.

An important contribution of this work is the experimental validation of the amplitude encoding stage through a reproducible simulation framework implemented in Qiskit. The obtained results establish a quantitative baseline that can support future investigations of more advanced quantum estimation and post-processing techniques.

The present investigation was limited to ideal single-qubit simulations and therefore does not represent the complete behavior of practical noisy quantum hardware. Future research may extend this methodology to noisy simulation models, multi-qubit systems, and more sophisticated quantum estimation algorithms to further evaluate the applicability of component-wise validation strategies in practical quantum computing.

Overall, the findings demonstrate that accurate amplitude encoding and systematic experimental validation provide a solid foundation for the study of practical quantum estimation workflows and support the continued development of hybrid quantum-classical methodologies for quantum computing applications.

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