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Original ـrticle

New Fixed-Point Theorem for (μ,β)-Convex Contractions in Complete b-Metric Spaces

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Abstract
Several fixed-point theorems for convex contraction mappings in b-metric spaces have been established in this paper. Additionally, we have provided some fixed-point results and introduced the notion of (μ,β)−convex contraction mapping in b-metric spaces.
Keywords. Fixed Point Theorem, (μ,β)-Convex Contraction, b-Metric Space, Complete Extended b-Metric Space, Istrățescu Convex Contraction.

Introduction

The theory of fixed points constitutes a pivotal and profoundly significant domain of inquiry within the realms of nonlinear and functional analysis, as it furnishes some of the most instrumental methodologies for addressing a variety of challenges encountered in applied sciences and engineering, including but not limited to dynamical systems, game theory, optimization theory, and the establishment of solutions pertaining to integral, differential, and matrix equations. Among the most consequential findings within fixed point theory is the Banach-Picard-Caccioppoli contraction principle [1,2]. This principle asserts that a mapping S defined on a complete metric space G that maps onto itself possesses a unique fixed point, contingent upon the stipulation that S qualifies as a contractive mapping, namely, there exists a constant $K \in [0,1)$ such that

$$d_\theta(S^2(h), S^2(g), a) \leq k d_\theta(h, g, a)$$

for all $h, g, a \in G$In light of its profound significance, this result has been the subject of extensive study and subsequent generalization.

In a broad sense, the generalization of fixed-point theorems can be approached in two principal ways. The first approach involves the relaxation of the conditions governing the contraction, while the second approach entails the generalization of the foundational space (which is commonly achieved by modifying the triangle inequality).

In the year 1982, Istrățescu [3] pioneered the classification of convex contractive mappings through the introduction of various convexity conditions, thereby yielding new generalizations of the contraction principle. Subsequently, convex contractions have also been subjected to further generalizations.

In recent years, an array of generalizations of classical metric spaces has been proposed. For example, Bakhtin [4] and Czerwik [5] have put forth the notion of b-metric spaces. In the year 2017, Kamran et al. [6] delineated the concept of extended b-metric spaces, thereby advancing the generalization of b-metric spaces.

for these kinds of contractions. Additionally, a few examples are provided to bolster our theoretical findings.

Definition 1.1: [1,2] Consider a nonempty set G and A mapping $S: G \to G$.A point is referred to as a fixed point of S if it satisfies $S(g) = g$. The set consisting of all fixed points of the mapping S is represented by Fix(S)

Definition 1.2: [2,7] Let G be a non-empty set and $d_\theta: G \times G \times G \to \mathbb{R}$ be function,such that:

1. $d_\theta(h, g) \geq 0$ for all $h, g \in G$,
2. $d_\theta(h, g) = 0$ iff $h = g$,
3. $d_\theta(h, g) = d_\theta(g, h)$ for all $h, g \in G$,
4. $d_\theta(h, g) \leq d_\theta(h, w) + d(w, g)$.

Then d_θ is called a metric on G and (G, d_θ) is called a metric space.

Example 1.1: [9] Let $G = \mathbb{R}$ and $S: G \times G \times G \to [0, \infty)$ be defined following:

$$d_\theta(h, g) = \begin{cases} e^{|h-g|} & , h \neq g \\ 0 & , h = g \end{cases}$$

Then ,is metric on S. Suoppes that $h = 1$, $g = 3$ and $w = 3$, since

$$d_\theta(1,3) = e^{|1-3|} = e^2 \approx 7.389 \geq d_\theta(1,2) + d_\theta(2,3) = e^{|1-2|} + e^{|2-3|} = 2e \approx 5.436$$

Then $7.389 \geq 5.436$ Then G is not a metric.

Definition 1.3: [2,7] Let $\{h_n\}_{n \in N}$ be a squence in an extended b_2-metric space (G, d_θ)

1) A sequence$\{h_n\}$ is a Cauchy sequence if and only if $d_\theta(h_n, h_m, a) \to 0$,when $n, m \to \infty$. for all $a \in G$.

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2) A sequence $\{h_n\}$ is convergent to $h \in X$, if for all $a \in G$, there exists $h \in G$, such that $\lim_{n\to\infty} d_\theta(h_n, h, a) = 0$.
3) An extended b –metric space (G, d_θ) is called complete if every Cauchy sequence is convergent sequeunce.

Definition 1.4: [7,8] Let G be a nonempty set and $\theta: G \times G \to [1, \infty)$ .A function $d_\theta: G \times G \to [0, \infty)$ is called an extended b-metric if for all $h, g, w \in G$. it satisfies:
$d_\theta(g, h) = 0 \Leftrightarrow g = h$
$d_\theta(g, h) = d_\theta(h, g)$;
$d_\theta(h, w) \leq \theta(h, w)[d_\theta(h, g) + d_\theta(h, w)]$
The pair (G, d_θ) is called an extended b-metric space.
Remark. 1.If $\theta(h, g) = s$ for $s \geq 1$ then we obtain the definition of a b-metric space.
Example 1.2. (kamran et al., 2017) Let $G = C([a, b], R)$ be the space of all continuous real-valued functions defined on $[a, b]$.
Define $\theta: h^2 \to [1, \infty)$ by

$$\theta(h, g) = |h(t) + g(t)| + 2$$

And $d_\theta: h^2 \to [0, \infty)$ by

$$d_\theta(h, g) = SUP_{t\in[a,b]}|h(t) + g(t)|^2$$

Then d_θ is called an extended b-metric and (G, d_θ) is a complete extended b-metric space.
Definition 1.5: [7,13] Let G be a nonempty set and $\theta: G \times G \times G \to [1,\infty)$ be mapping. Afunction $d_\theta: G \times G \times G \to [0,\infty)$ is an extended b_2- Metric on G if for all $a, h, g, z \in G$ the following conditions hold:
1) For every pari of distinct points $h, g \in G$,there exists apint $z \in G$ such that $d_\theta(h, g, z) \neq 0$.
2) If at least two of three pointsh, g, z are the same then $d_\theta(h, g, z) = 0$.
3) The symmetry: $d_\theta(h, g, z) = d_\theta(h, z, g) = d_\theta(g, h, z) = d_\theta(g, z, h) = d_\theta(z, h, g) = d_\theta(z, g, h)$ for all h, g, z ∈ G.
4) The rectangle inequality: $d_\theta(h, g, z) \leq \theta(h, g, z)[d_\theta(h, g, a) + d_\theta(g, z, a) + d_\theta(z, h, a)$ for all $h, g, z, a \in G$.
The d_θ is called an extended b_2 – Metric on G and the pair (G, d_θ) is called an extended b_2-Metric space.

Main Results

Definition 2.1: [8,9,10] Let (G, d_θ) be a complete metric space . A map $S: G \to G$ known as Is tratescu convex contraction if
$d_\theta(S^2(h), S^2(g), a) \leq a_1 d_\theta(S(h), S(g), a) + b_1 d_\theta(S(h), S(g), a)$ for any $h, g, a \in G$
where a_1 and b_1 are constants which satisfy $0 < a_1$ and $a_1 + b_1 < 1$
Remark 2.1.
i) If $a_1 = 0$, the convex contraction condition reduces to the classial " asymptotic" cotraction:

$$d_\theta(S^2(h), S^2(g), a) \leq b_1 d_\theta(h, g, a),$$

ii) If $b_1 = 0$, the convex contraction reduces to the well. Known Banach contraction condition:

$$d_\theta(S(h), S(g), a) \leq a_1 d_\theta(h, g, a) \ \text{for all} \ h, g, a \in G$$

Subject to a change of notation.
Definition 2.2: [8,10] Let S be a mapping from a metric space G into itself. The set

$$\mathcal{O}(S, g) = \{S^n g: n = 0,1,2, \dots\}$$

Is referred to as the orbit of S starting at g. We say that S is orbitally continuous at a point $z \in G$ if for every sequence
$\{g_n\} \subset \mathcal{O}(S, g)$, with $g \in G$, the condition

$$\lim_{n\to\infty} g_n = z \implies \lim_{n\to\infty} S(g_n) = S(z).$$

Definition 2.3: [3,10,11] Let (G, d_θ) denote a complete metric space. A continuous mapping $S: G \to G$ is termed a Chatterjea two-sided convex contraction if there are constants $a_1, a_2, b_1, b_2 \in (0,1)$ and the subsequent inequality is satisfied:

$$d_\theta(S^2(h), S^2(g), a) \leq a_1 d_\theta(h, S(g), a) + a_2 d_\theta(S(g), S(g), a) + b_1 d_\theta(S(g), S(h), a) + b_2 d_\theta(S(h), S(h), a)$$

For any $h, g, a \in G$ and $a_1 + a_2 + b_1 + b_2 < 1$.
Definition 2.4: [10,8,12] Let (G, d_θ) be a metric space and a mapping $S: G \to G$ is said to be a convex contraction of order $k \in N$ if there exist $a_0, a_1, \dots, a_{k-1} \in (0,1)$, with $a_0 + a_1 + \dots + a_{k-1} < 1$ such that , for all $h, g, a \in G$ the following inequality.

$$d_\theta(S^{[k]}h, S^{[k]}g, a) \leq a_0 d_\theta(h, g, a) + a_1 d_\theta(Sh, Sg, a) + \dots + a_{k\pi-1} d_\theta(S^{[k-1]}h, S^{[k-1]}g, a).$$

Definition 2.5: [10,14,16] A mapping G on a metric space (G, d_θ) is said to be weakly contractive mapping if for all $h, g, a \in G$

$$d_\theta(Sh, Sg, a) \leq d_\theta(h, g, a) - \beta(d_\theta(h, g, a))$$

Where $\beta: [0, \infty) \to [0, \infty)$ is a continuous and non-decreasing function such that $\beta(t) = 0 \Leftrightarrow t = 0$.
If one takes $\beta(t) = (1 - \lambda)t$, where $0 \leq \lambda < 1$, then weak contraction [2] reduces to a Banach contraction (1).

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Theorem 2.1: [8] If G is a weakly contractive mapping on a complete metric space (G, d_θ) then G has a unique fixed point.
Lemma 2.2: [14,15] Let(G, d_θ) be a complete b-metric space and let $S: G \to G$ be an orbitally continuous mapping. If the picard iterations sequence basedis based on an initial point $h_0 \in G$, i.e., $h_n = S^n h_0$, for all $n \geq 0$, is a Cauchy sequence, then G has a fixed point.
Theorem 2.3: [17] Let (G, d_θ) be a complete metric space and let
$S: G \to G$ be a self-mapping satisfying the inequality.

$$\mu(d_\theta(Sh, Sg, a) \leq \mu(d_\theta(h, g, a) - \beta\big(d_\theta(h, g, a)\big),$$

Where $\mu, \beta: [0, \infty) \to [0, \infty)$ are both continuous and monotonic non decreasing functions with $\mu(t) = 0 = \mu(t)$ if and anly if $t = 0$. Then G has a unique fixed point.
Definition 2.6: [10,17] Let (G, d_θ) be a metric space and a mapping $S: G \to G$ is said to be a generalize (μ, β)-convex contraction mapping of order $m \in N$ if there exist two continuos and monotonic non decreasing functions $\mu, \beta: [0, \infty) \to [0, \infty)$ such that $\mu(t) = 0 = \beta(t)$ if and only if $t = 0$ and satisfies the following inequality.

$$\mu(d_\theta\big(S^{[k]}h, S^{[k]}g, a\big)) \leq \mu(M_k(h, g, a) - \beta\big(N_k(h, g, a)\big), \qquad (2.1)$$

For all $h, g, a \in G$, where

$$M_m(h, g, a) = \max\{d_\theta(h, g, a), d_\theta(h, g, a), \ldots, d_\theta(S^{[k-1]}h, S^{[k-1]}g, a)\}$$

and

$$N_n(h, g, a) = \min\{d_\theta(h, g, a), d_\theta(h, g, a), \ldots, d_\theta(S^{[k-1]}h, S^{[k-1]}g, a)\}$$

Now we state our main result.
Theorem 2.4: [8,10,14,16] Every (μ, β)-weak convex contraction mapping of order $m \in \mathbb{N}$ in a complete metric space has a unique fixed point.
Proof:
Let h_0 be an arbitrary point in G . We construct a sequence$\{h_n\}$ by
$h_n = S^{[n]}h_{0,}$ $n = 1,2, \ldots$ If $h_n = h_{n+1}$ for some $n \in \mathbb{N}$ then h_n is a fixed point of G .Thus we suppose that $h_n \neq h_{n+1}$ for all $n \in \mathbb{N}$ and $d_\theta(h_{n-1}, h_n, a) > 0$ for all $n \in \mathbb{N}$. Substituting $h = S^{[n]}h_0$ and $g = S^{[n+1]}h_0$ in (2.1), we get

$$\mu(d_\theta(h_{m+k}, h_{m+k+1}, a)) \leq \mu(M_k(h_n, h_{n+1}, a) - \beta\big(N_k(h_n, h_{n+1}, a)\big) \qquad (2.2)$$

Where

$$M_k(h_n, h_{n+1}, a) = \max\{d_\theta(h_n, h_{n+1}, a), d_\theta(h_{n+1}, h_{n+2}, a), \ldots, d_\theta(h_{n+k-1}, h_{n+k}, a)\}$$

and

$$N_n((h_n, h_{n+1}, a) = \min\{d_\theta(h_n, h_{n+1}, a), d_\theta(h_{n+1}, h_{+2}, a), \ldots, d_\theta(h_{n+k-1}, h_{n+k}, a)\}$$

From (2.2) ,we get

$$\mu(d_\theta(h_{m+k}, h_{m+k+1}, a) \leq \mu(M_k(h_k, h_{m+1}, a)) \qquad (2.3)$$

which implies

$$d_\theta(h_{m+k}, h_{m+k-1}, a) \leq M_k(h_n, h_{m+1}, a) \qquad (2.4)$$

It follows that the both sequences $\{d_\theta(h_m, h_{m+1}, a)\}_{n \in \mathbb{N}}$ and $\{M_m(h_m, h_{m+1}, a)\}_{n \in \mathbb{N}}$ are monotonic decreasing and therefore there exists $r \geq 0$ such that

$$d_\theta(h_m, h_{m+1}, a) \to r \; as \; m \to \infty. \qquad (2.5)$$

Making $m \to \infty$ in(2.5), we get

$$\mu(r) \leq (\mu(r) - \beta(r), a), \qquad (2.6)$$

Which is a contradiction unless $r = 0$. Hence

$$\lim_{m \to \infty} d_\theta\,(h_m, h_{m+1}, a) = 0 \; for \; all \; m \in \mathbb{N} \cup \{0\}.$$

Now we will prove that$\{h_n\}$ is a Cauchy sequence.If possible let $\{h_n\}$ is not a Cauchy sequence, than for each positive integer n, there exists $\epsilon > 0$ and subsequences $\{h_{p(n)}\}$ and $\{h_{q(n)}\}$ of $\{h_{n(n)}\}$ with $k < p(k) < q(k)$ such that

$$d_\theta\big(h_{p(n)}, h_{q(n)}, a\big) \geq \epsilon. \qquad (2.7)$$

Let $q(k)$ be the least positive integer exceeds $p(k)$ and satisfies [8], for each positive integer n, then it clear that

$$\epsilon \leq d_\theta\big(h_{p(n)}, h_{q(n)}, a\big) \leq d_\theta\big(h_{p(n)}, h_{q(n)-1}, a\big) + d_\theta\big(h_{q(n)-1}, h_{q(n)}, a\big)$$
$$< \epsilon + d_\theta\big(h_{q(n)-1}, h_{q(n)}, a\big).$$

Making $n \to \infty$ and using (2.7)

$$\lim_{n \to \infty} d_\theta\big(h_{p(n)}, h_{q(n)}, a\big) = \epsilon \qquad (2.8)$$

again
$d_\theta\big(h_{q(n)-1}, h_{p(n)-1}, a\big) \leq$

$$d_\theta\big(h_{q(n)-1}, h_{q(n)}, a\big) + d_\theta\big(h_{p(n)}, h_{q(n)}, a\big) + d_\theta\big(h_{p(n)}, h_{p(n)-1}, a\big)$$

making $n \to \infty$ in the above inequality and using (2.7),(2.8),we get

$$\lim_{n \to \infty} d_\theta\big(h_{p(n)-1}, h_{q(n)-1}, a\big) = \epsilon \qquad (2.9)$$

Similarly we can prove that

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$$\lim_{n\to\infty} d_\theta\left(h_{p(n)-m}, h_{q(n)-m}, a\right) = \epsilon \ , m \in N \qquad (2.10)$$

Now putting and in $h = h_{p(k)-m}$ and $g = h_{q(k)-m}$ in (2.5) using (2.8) we obtain

$\mu(\epsilon) \le \mu(d_\theta\left(h_{p(n)}, h_{q(n)}, a\right) \le \mu(M_k\left(h_{p(n)-k}, h_{q(n)-k}, a\right) - \beta(N_k\left(h_{q(n)-k}, h_{q(n)-k}, a\right)$

Making $n \to \infty$ and utilizing (2.9) and (2.10),we obtain

$$\mu(\epsilon) \le (\mu(\epsilon) - \beta(\epsilon), a), \qquad (2.11)$$

which is a contradiction if $\epsilon > 0$. This shows that $\{h_n\}$ is a Cauchy sequence and hence it is convergent in the complete metric space G .Let

$$\{h_n\} \to z \quad \text{as} \quad n \to \infty$$

Substituting $h = h_n$ and $g = w$, we obtain

$$\mu(d_\theta(h_{m+n}, Sw) \le \mu(M_m(h_n, w) - \beta\left(N_m(h_n, w)\right).$$

making $n \to \infty$ and using continuity of μ and β ,we have

$$\mu(d_\theta(w, Sw) \le \mu(0) - \beta(0) = 0,$$

which implies $\mu\left(d_\theta(w, Sw)\right) = 0$, that is,

To prove the uniqueness of the fixed point, let us suppose that w_1 and w_2 are two fixed points of G.putting $h = w_1$ and $g = w_2$ in (2.5),we get

$$\mu(d_\theta\left(S^{[k]}w_1, S^{[k]}w_2, a\right)) \le \mu(M_k(w_1, w_2, a) - \beta\left(N_k(w_1, w_2, a)\right), \quad (2.12)$$

where

$$M_k(w_1, w_2, a) = \max\left\{d_\theta(h, g, a), d_\theta(h, g, a), \dots, d_\theta(S^{[k-1]}h, S^{[k-1]}g, a)\right\}$$

and

$$N_k(w_1, w_2, a) = \min\left\{d_\theta(h, g, a), d_\theta(h, g, a), \dots, d_\theta(S^{[k-1]}h, S^{[k-1]}g, a)\right\}$$

implies

$$\beta(d_\theta(w_1, w_2, a) \le 0,$$

from(2.9)

$$\mu(d_\theta(w_1, w_2, a) \le \mu(d_\theta(w_1, w_2, a) - \beta(d_\theta(w_1, w_2, a)$$

Or equivalently $d_\theta(w_1, w_2, a) = 0$ that is $w_1 = w_2$. The proves the uniqueness of the fixed point.

Corollary 3.2: [8,10] Let (G, d_θ) be a complete extended b-metric space and let $S: G \to G$ be an orbitally continuous mapping. If there exists $k \in [0,1)$ such that the inequality

$$d_\theta(S^2h, S^2g, a) \le k d_\theta(Sh, Sg, a)$$

Holds for any $h, g, a \in G$, and if there exist $h_0 \in G, \beta < \frac{1}{\sqrt{k}}$, , and $n_0 \in \mathbb{N}$such that the inequality

$$\prod_{i=n}^{j} \theta(h_i, h_{n+p}, a) \le \beta^{j-n+1}$$

Holds for all $n \le n_0, p \ge 1$, and $j \in \{n, n+1, \dots, n+p-1\}$, where $h_n = S^n h_0, n \ge 0$, then S has a unique fixed point .

Example 2.1. Let $G = \{0,1,2\}$ be endowed with b-metric give in Example 1.1

Define : $S: G \to G$ by $S0 = S2 = 0$ and $S1 = 2$ if $h = 0$ and $g = 1$, then for all $\lambda \in (0,1)$ we have

$$d_\theta(S0, S1, a) = d_\theta(0,2) = e^2 > \lambda e = \lambda\, d_\theta(0,1)$$

Hence, S does not satisfy the condition of the Banach contraction principle [7]. Since $S^2h = 0$ for all $h \in G$, then all assumptions of Theorem 2.2 are satisfied. Hence S has a unique.

Now, we give the definitions of (μ, β)-admissible convex contraction of order 2 and two-sided convex contraction mappings in the setting of b-metric space and prove several fixed-point theorems for such mappings on b-metric spaces.

Corollary 2.5: [16] (Istrățescu is fixed point theorem convex contraction). Let $S: G \to G$ is a convex contraction of order $m \in \mathbb{N}$ is complete metric space (G, d_θ) then there exists afixed point of S and this is unique.

Proof.

$$M_k(w_1, w_2, a) = \max\left\{d_\theta(h, g, a), d_\theta(h, g, a), \dots, d_\theta(S^{[k-1]}h, S^{[k-1]}g, a)\right\}$$

and

$$N_n(h, g, a) = \min\left\{d_\theta(h, g, a), d_\theta(h, g, a), \dots, d_\theta(S^{[k-1]}h, S^{[k-1]}g, a)\right\}$$

Now we have given that S is convex contraction of order m on complete metric space (G, d_θ) so there exist positive numbers $a_0, a_1, \dots, a_{m-1} \in (0,1)$ such that

$a_1 + a_2 + \dots + a_{nm} < 1$ and for all $g, h \in S$

$$\mu\left(d_\theta\left(S^{[k]}h, S^{[k]}g, a\right)\right) \le a_0 d_\theta(h, g, a) + a_1 d_\theta(Sh, Sg, a) + \dots + a_{n-1} d_\theta\left(\left(S^{[k]}h, S^{[k]}g, a\right)\right)$$
$$\le (a_1 + a_2 + \dots + a_{m-1}) M_m(h, g, a)$$
$$= (a_1 + a_2 + \dots + a_{m-1}) M_m(h, g, a) + M_m(h, g, a) - M_m(h, g, a)$$

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$$= M_m(h,g,a) - (1 - (a_1 + a_2 + \cdots + a_{n-1})M_m(h,g,a)$$

Note that $\mu(r) = (1 - (a_1 + a_2 + \cdots + a_{n-1}))\,\beta(r)$ and $\beta(r) = r$
for all $r \in [0, \infty)$ we have.

$$\mu\big(d_\theta\big(S^{[k]}h, S^{[k]}g, a\big)\big) \le \mu(M_k(h,g,a) - \beta\big(N_k(h,g,a)\big), \qquad (2.13)$$

An we know that $N_n(h,g,a) \le M_n(h,g,a)$ for $n \in \mathbb{N}$ so

$$\mu(N_n(h,g,a)) \le \mu(M_n(h,g,a))$$

Inequality and from (2.13) we have

$$\mu(d_\theta\big(S^{[k]}h, S^{[k]}g, a\big)) \le \mu(M_k(h,g,a) - \beta\big(N_k(h,g,a)\big)$$

We get S has a unique fixed point.

Conclusion

This manuscript has effectively formulated innovative fixed point theorems pertaining to convex contraction mappings within the framework of complete b-metric spaces. The principal contribution lies in the introduction of the convex contraction mapping alongside the derivation of fixed point results associated with this emergent category of contractions. We have demonstrated that every weak convex contraction mapping of order 2 within a complete b-metric space is endowed with a unique fixed point (Theorem). This finding serves to generalize various classical fixed-point theorems, encompassing Istrăţescu's convex and weakly contractive mappings. The conclusions were substantiated by precise definitions and a thorough proof of uniqueness. This research augments the existing literature on metric fixed point theory, furnishing instruments for subsequent inquiries into generalized metric spaces.

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