Alqalam Journal of Medical and Applied Sciences. 2026;9(6):1540-1547
https://doi.org/10.54361/ajmas.269613

Original article

The Solution of Two-Dimensional Time-Fractional Order Reaction-Diffusion Model by Using the Natural Transform Approach

Fatma Ahmed[1]*, Muna Mansour[1], Mahmoud El-Horbaty[2]

[1]*Department of Mathematics, Faculty of Sciences -Alajelat, University of Zawia, Alajelat, Libya*
[2]*Department of Mathematics, Faculty of Sciences, Zagazig University, Zagazig, Egypt*
Corresponding Email. *f.ahmed@zu.edu.ly*

Abstract
In this article, the method of Natural transform is coupled with Adomian's polynomials and employed to successfully solve the two-dimensional time-fractional order Fisher equation in Caputo's sense, which is well known for modeling reaction-diffusion problems. The reliability and proficiency of the method are presented using tables and illustrations for arbitrary values of the fractional order. Indeed, the results have proved the effectiveness of the method in solving multi-dimensional fractional order differential equations.
Keywords. Natural Transform, Adomian Polynomials, Reaction-diffusion Equation, Caputo Derivative**.**

Introduction

Fractional calculus (FC) has been the center of attraction of modern studies in diverse branches of science, due to its vital applications in modeling real-world problems with heritage properties. Moreover, the fractional order derivative has the unique ability to apprehend particular features in the diffusion mechanism behavior of complex media [1,2].

In recent decades, tremendously effective techniques involving integral transforms have been introduced and adopted to construct the analytical and semi-analytical solutions of both fractional partial differential equations (FPDEs) and partial differential equations (PDEs), notably, the Adomian decomposition method (ADM), Homotopy Perturbation method (HPM), Homotopy Analysis method (HAM), the Residual Power Series method (RPSM), Variational iteration method (VIM), and the General Transform Adomian Decomposition Method (GTADM) [3–17]. The Tendency towards coupling these methods with integral transforms, including the Laplace transform and the Natural Transform (NT), has gained great attention in recent literature [18–33]. In particular, the Natural decomposition method (NDM), which was first established by Rawashdeh and Maitama in 2014 [34], has been utilized extensively to solve FPDEs. Further, Rawashdeh [35] has proved some NT theorems and properties, validating the effectiveness of the NDM on the Harry Dym equation and the nonlinear time-fractional Fisher's equation in one dimension. Additionally, the One-dimensional time-fractional generalized Fisher equation has been studied and analyzed using NDM.

The validation of the scheme was presented via numerical results, yet proven to be a trustworthy method in solving such problems [36]. On the other hand, Usman et al. [37] applied the NDM to solve fuzzy fractional nonlinear generalized Fisher equations; the series solution was rapidly accurate, plus it has been confirmed to be a time-efficient technique. In an article done by Veeresha and Prakasha [38] a two-dimensional fractional order biological model was numerically simulated using NDM for various fractional orders, eventually affirming the proficiency and reliability of the method in solving multi-dimensional FPDEs. Moreover, two interesting studies done by Baranwal et al. and Tchier et al. [39,40] have investigated the solution of the two-dimensional time fractional Fisher equation utilizing an integration of VIM, ADM, and RPSM, respectively. These methods were tested to provide a numerical simulated proof of rapid convergent series solutions.

Motivated by the research work, we aimed, in this article, to investigate the series solution of the two-dimensional time-fractional Fisher equation by applying the NDM. The article is organized to recall important basic concepts of FC and NT in Section 2. The methodology of the fractional NDM is discussed in detail in Section 3. A fully detailed numerical simulation of the solution of the time-fractional Fisher equation in two dimensions is illustrated via 2D and 3D profiles were presented in Section 4, in addition to the absolute error behavior on various values of time. In Section 5, concluding remarks on the results and findings are given in brief.

Preliminaries

This section recalls the essential definitions and properties of FC and NT [1,2,41–45].

Definition 1. [1,2]

Let $g(\tau), \tau > 0$ be a function in $L([0,1], \mathbb{R})$ that is sufficiently smooth on $\mathbb{C}_{-1}^{n}$, for $n \in \mathbb{N} \cup \{0\}$, then the fractional order γ of Caputo's derivative of $\mathfrak{g}(\tau)$ is:

Received*: 08-04-2026 -* ***Accepted****: 06-06-2026 -* ***Published****: 11-06-2026*

https://doi.org/10.54361/ajmas.269613

$$\mathfrak{D}_\tau^\gamma \mathrm{g}(\tau)=\begin{cases}\mathfrak{I}^{n-\gamma}\mathrm{g}^n(\tau) & ,\text{if } n-1<\gamma<n, n\in\mathbb{N}\\ \dfrac{d^n\mathrm{g}(\tau)}{d\tau^n} & ,\gamma=n\end{cases} \tag{1}$$

where, $\gamma>0$, $\mathfrak{I}^{n-\gamma}\mathrm{g}^{(n)}(\tau)=\frac{1}{\Gamma(n-\gamma)}\int_0^\tau(\tau-\varepsilon)^{n-\gamma-1}\mathrm{g}^{(n)}(\varepsilon)d\varepsilon$ is the Riemann-Liouville (R-L) integral of $\mathrm{g}^{(n)}(\tau)$, and $\Gamma(\varepsilon)=\int_0^\infty e^{-s}s^{\varepsilon-1}d\tau, \varepsilon\in\mathbb{C}$ is the Gamma function.

Definition 2. [1,2,43]
Let $\mathfrak{u}(x,\tau)$ be a multi-variable function, then the fractional order γ of Caputo's fractional derivative is given by:

$$\mathfrak{D}_\tau^\gamma \mathfrak{u}(x,\tau)=\frac{\partial u(x,\tau)}{\partial\tau^\gamma}=\begin{cases}\dfrac{1}{\Gamma(m-\gamma)}\displaystyle\int_0^\tau(\tau-\varepsilon)^{m-\gamma-1}\frac{\partial^m\mathfrak{u}(x,\varepsilon)}{\partial\varepsilon^m}\,d\varepsilon, & m-1<\gamma<m\\ \dfrac{\partial^m\mathfrak{u}(x,\tau)}{\partial\tau^m}, & \gamma=m\in\mathbb{N}\end{cases} \tag{2}$$

Definition 3. [1,2,43]
The two parameter Mittag-Leffler function has the infinite series form:

$$E_{\gamma,\beta}(\tau)=\sum_{k=0}^\infty\frac{\tau^k}{\Gamma(k\gamma+\beta)} \qquad ,\gamma,\beta>0 \text{ and } \tau\in\mathbb{C} \tag{3}$$

and the following is satisfied:
If $\beta=1$ then $E_\gamma(\tau)=\sum_{k=0}^\infty\frac{\tau^k}{\Gamma(k\gamma+1)}$.
If $\gamma=1$ and $\beta=1$ then $E(t)=\sum_{k=0}^\infty\frac{\tau^k}{k!}=e^\tau$,
which is the classical exponential function.

Definition 4.[41,42]
The Natural Transform (NT) of a function $\mathrm{g}(x_i,\tau)$, is defined as:

$$\mathbb{N}^+[\mathrm{g}(x_i,\tau)] = \int_0^\infty e^{-s\tau}\mathrm{g}(x_i,u\tau)d\tau=\frac{1}{u}\mathcal{L}\{\mathrm{g}(x_i,\tau)\}=\mathfrak{G}(x_i,s,u) \qquad ,s,u\in\mathbb{R}^+ \tag{4}$$

where s and u are the NT parameters, noting that if $u=1$, we have $\mathbb{N}^+[\mathrm{g}(x_i,\tau)]=\mathfrak{G}(x_i,s,1)=\int_0^\infty e^{-s\tau}\mathrm{g}(x_i,\tau)d\tau=\mathcal{L}\{\mathrm{g}(x_i,\tau)\}=\mathfrak{G}(x_i,s)$ and it is the Laplace Transform of $\mathrm{g}(x_i,\tau)$.

Definition 5. [41,42]
The Inverse NT of Eq. (4) is given by:

$$\mathbb{N}^-[\mathfrak{G}(x_i,s,u)]=\frac{1}{2\pi i}\int_{q-i\infty}^{q+i\infty}e^{-s\tau}\mathfrak{G}(x_i,s,u)ds=\mathrm{g}(x_i,\tau) \tag{5}$$

Where $q\in\mathbb{R}$ such that $s=x+iy$ in the complex plane.
Lemma 1. [41,42]
The NT of Caputo fractional derivative of a function $\mathrm{g}(x_i,\tau)$ with respect to the variable τ is defined as:

$$\mathbb{N}^+\left[\mathfrak{D}_\tau^\gamma\mathrm{g}(x_i,\tau)\right]=\frac{s^\gamma}{u^\gamma}\mathfrak{G}(x_i,s,u)-\sum_{k=0}^{n-1}\frac{s^{\gamma-(k+1)}}{u^{\gamma-k}}\left[\lim_{\tau\to0}\mathfrak{D}_\tau^\gamma\mathrm{g}(x_i,\tau)\right] \tag{6}$$

Lemma 2. [41,42]
The NT of the fractional derivative of a function $\mathrm{g}(x_i,\tau)$ with respect to the variable x is given by:

$$\mathbb{N}^+\left[\mathfrak{D}_x^\gamma\mathrm{g}(x_i,\tau)\right]=\mathfrak{D}_x^\gamma\mathfrak{G}(x_i,s,u) \tag{7}$$

The Fractional Natural Decomposition Method (NDM)

According to Rawashdeh [35] and Belgacem et al. [41], and by considering the following multi-dimensional FPDs system of equations:

$$\mathfrak{D}_\tau^\gamma v_i(\mathbf{X},\tau)+\mathbf{R}v_i(\mathbf{X},\tau)+\mathbf{N}v_i(\mathbf{X},\tau)=\mathrm{g}_i(\mathbf{X},\tau) \tag{8}$$

$m-1\le\gamma\le m,\quad m\in\mathbb{N},\ i=1,2,\dots n$.
with the IC of the form:

$$\mathfrak{D}_\tau^\gamma v_i(\mathbf{X},0)=\varphi_{ik_i}(\mathbf{X}) \tag{9}$$

$m-1\le\gamma\le m,\quad m\in\mathbb{N},\ i=1,2,\dots n$.
where $\mathfrak{D}_\tau^\gamma=\frac{\partial^\gamma}{\partial\tau^\gamma}$ is the Caputo fractional derivative of order γ $(m-1\le\gamma\le m)$, $\mathrm{X}=(x,y,z)^T$, $\mathrm{R}v_i(\mathrm{X},\tau)$ and $\mathrm{N}v_i(\mathrm{X},\tau)$ are the operators of linearity and nonlinearity (respectively). $\mathrm{g}_i(\mathrm{X},\tau)$ is a continuous function

***Received**: 08-04-2026 - **Accepted**: 06-06-2026 - **Published**: 11-06-2026*

https://doi.org/10.54361/ajmas.269613

representing the source term. The methodology of NDM consists of applying the NT on both sides of Eq. (8), yielding:

$$\mathbb{N}^{+}\left[\mathfrak{D}_{\tau}^{\gamma}\mathcal{v}_i(\mathrm{X},\tau)\right]+\mathbb{N}^{+}[\mathrm{R}\mathcal{v}_i(\mathrm{X},\tau)+\mathrm{N}\mathcal{v}_i(\mathrm{X},\tau)]=\mathbb{N}^{+}[\mathfrak{g}_i(\mathrm{X},\tau)\ ] \quad (10)$$

Using the differentiation property of the NT, we have:

$$\mathbb{N}^{+}\left[\mathfrak{D}_{\tau}^{\gamma}\mathcal{v}_i(\mathrm{X},\tau)\right]=\frac{\varphi_{ik_i}(\mathrm{X})}{s}+\frac{u^{\gamma}}{s^{\gamma}}\mathbb{N}^{+}[\mathfrak{g}_i(\mathrm{X},\tau)\ ]-\frac{u^{\gamma}}{s^{\gamma}}\mathbb{N}^{+}[\mathrm{R}\mathcal{v}_i(\mathrm{X},\tau)+\mathrm{N}\mathcal{v}_i(\mathrm{X},\tau)] \quad (11)$$

Operating with the inverse NT on both sides of Eq. (11), give rise to:

$$\mathcal{v}_i(\mathrm{X},\tau)=\varphi_{ik_i}(\mathrm{X})+\mathbb{N}^{-}\left[\frac{u^{\gamma}}{s^{\gamma}}\mathbb{N}^{+}[\mathfrak{g}_i(\mathrm{X},\tau)\ ]\right]-\mathbb{N}^{-}\left[\frac{u^{\gamma}}{s^{\gamma}}\mathbb{N}^{+}[\mathrm{R}\mathcal{v}_i(\mathrm{X},\tau)+\mathrm{N}\mathcal{v}_i(\mathrm{X},\tau)]\right] \quad (12)$$

Now the method suggests the solution of Eq. (12), to have the infinite series form:

$$\mathcal{v}_i(\mathrm{X},\tau)=\sum_{n=0}^{\infty}\mathcal{v}_{in}(\mathrm{X},\tau) \quad (13)$$

And the nonlinear term $\mathrm{N}\mathcal{v}_i(\mathrm{X},\tau)$ is decomposed as infinite series of the form:

$$\mathrm{N}\mathcal{v}_i(\mathrm{X},\tau)=\sum_{n=0}^{\infty}\mathfrak{A}_{in} \quad (14)$$

Where $\mathfrak{A}_{in}$ is the Adomian polynomials [46]. In view of Eq. (13) and Eq. (14) the solution of $\mathcal{v}_i(\mathrm{X},\tau)$ Eq. (12) can be rewritten as:

$$\sum_{n=0}^{\infty}\mathcal{v}_{in}(\mathrm{X},\tau)\ =\varphi_{ik_i}(\mathrm{X})+\mathbb{N}^{-}\left[\frac{u^{\gamma}}{s^{\gamma}}\mathbb{N}^{+}[\mathfrak{g}_i(\mathrm{X},\tau)\ ]\right]-\mathbb{N}^{-}\left[\frac{u^{\gamma}}{s^{\gamma}}\mathbb{N}^{+}\left[\mathrm{R}\sum_{n=0}^{\infty}\mathcal{v}_{in}(\mathrm{X},\tau)+\mathrm{N}\sum_{n=0}^{\infty}\mathfrak{A}_{in}\right]\right] \quad (15)$$

Thus, the recurrence relation takes the form:

$$\mathcal{v}_{i0}(\mathrm{X},\tau)=\varphi_{ik_i}(\mathrm{X})+\mathbb{N}^{-}\left[\frac{u^{\gamma}}{s^{\gamma}}\mathbb{N}^{+}[\mathfrak{g}_i(\mathrm{X},\tau)\ ]\right] \quad (16\ a)$$

$$\mathcal{v}_{ij+1}(\mathrm{X},\tau)=-\mathbb{N}^{-}\left[\frac{u^{\gamma}}{s^{\gamma}}\mathbb{N}^{+}\left[\mathrm{R}\mathcal{v}_{ij}(\mathrm{X},\tau)+\mathfrak{A}_{in}\right]\right] \quad (16\ b)$$

Hence, the m^{th} approximate solution of the system in Eq. (8) is:

$$\mathcal{v}_i(\mathrm{X},\tau)=\sum_{n=0}^{m}\mathcal{v}_{in}(\mathrm{X},\tau)=\ \mathcal{v}_{i0}(\mathrm{X},\tau)+\mathcal{v}_{i1}(\mathrm{X},\tau)+\mathcal{v}_{i2}(\mathrm{X},\tau)+\cdots+\mathcal{v}_{im}(\mathrm{X},\tau) \quad (17)$$

eventually, the analytical solution is obtained, as $\lim_{m\to\infty}\mathcal{v}_i(\mathrm{X},\tau)=\mathcal{v}(\mathrm{X},\tau)$

Application, Results, and Discussion

In this section, we are investigating the two-dimensional time-fractional Fisher equation [40] , that is widely recognized for modeling a reaction-diffusion real-world phenomena. Consider the following initial value problem (IVP):

$$\mathfrak{D}_{\tau}^{\gamma}\mathcal{v}(x,y,\tau)=\frac{1}{2}\left(\partial_x^2\mathcal{v}(x,y,\tau)+\partial_y^2\mathcal{v}(x,y,\tau)\right)+\mathcal{v}^2(x,y,\tau)\left(1-\mathcal{v}(x,y,\tau)\right) \quad (18)$$

With the IC:

$$\mathcal{v}(x,y,0)=1/\left(1+e^{\frac{x+y}{\sqrt{2}}}\right) \quad (19)$$

where $\tau>0$, and the time-fractional order in the Caputo's approach is $\mathfrak{D}_{\tau}^{\gamma}$ $(0<\gamma\le 1)$. The FPDE (18) corresponds to the Natural equation:

$$\mathcal{v}(x,y,\tau)=\mathcal{v}(x,y,0)+\mathbb{N}^{-}\left[\frac{u^{\gamma}}{s^{\gamma}}\mathbb{N}^{+}\left[\frac{1}{2}\left(\partial_x^2\mathcal{v}(x,y,\tau)+\partial_y^2\mathcal{v}(x,y,\tau)\right)+\mathcal{v}^2(x,y,\tau)\left(1-\mathcal{v}(x,y,\tau)\right)\right]\right] \quad (20)$$

Following the decomposition procedure presented in Section 3 along with Eq. (15), Eq. (16 a) and Eq. (16 b), we obtain the recurrence relation of Eq. (20) as:

$$\mathcal{v}_0(x,y,\tau)=\mathcal{v}(x,y,0)=\varphi(x,y)$$

$$\mathcal{v}_{j+1}(x,y,\tau)=\mathbb{N}^{-}\left[\frac{u^{\gamma}}{s^{\gamma}}\mathbb{N}^{+}\left[\frac{1}{2}\left(\partial_x^2\mathcal{v}_j(x,y,\tau)+\partial_y^2\mathcal{v}_j(x,y,\tau)\right)+\mathfrak{A}_j\right]\right],j\ge 0 \quad (21)$$

where $\mathfrak{A}_j=\frac{1}{j!}\frac{d^j}{d\lambda^j}\left[\mathrm{N}\sum_{i=1}^{j}\lambda^i\mathcal{v}^i(x,y,\tau)\right]_{\lambda=0}$, and the subsequent few terms are:

***Received**: 08-04-2026 - **Accepted**: 06-06-2026 - **Published**: 11-06-2026*

https://doi.org/10.54361/ajmas.269613

$$\mathcal{v}_0(x,y,\tau)=\mathcal{v}(x,y,0)=\frac{1}{\mathrm{e}^{(x+y)/\sqrt{2}}+1}$$
$$\mathcal{v}_1(x,y,\tau)=\frac{\mathrm{e}^{(x+y)/\sqrt{2}}}{2\left(\mathrm{e}^{(x+y)/\sqrt{2}}+1\right)^2}\frac{\tau^{\gamma}}{\Gamma(\gamma+1)}$$
$$\mathcal{v}_2(x,y,\tau)=\frac{\mathrm{e}^{(x+y)/\sqrt{2}}\left(\mathrm{e}^{(x+y)/\sqrt{2}}-1\right)}{4\left(\mathrm{e}^{(x+y)/\sqrt{2}}+1\right)^3}\frac{\tau^{2\gamma}}{\Gamma(2\gamma+1)}$$
$$\mathcal{v}_3(x,y,\tau)=\frac{\mathrm{e}^{\frac{(x+y)}{\sqrt{2}}}\left(\Gamma(\gamma+1)^2\left(5\mathrm{e}^{\frac{(x+y)}{\sqrt{2}}}+\mathrm{e}^{\frac{3(x+y)}{\sqrt{2}}}-7\mathrm{e}^{\sqrt{2}(x+y)}+1\right)+2\,\Gamma(2\gamma+1)\,\mathrm{e}^{\frac{(x+y)}{\sqrt{2}}}\left(\mathrm{e}^{\frac{(x+y)}{\sqrt{2}}}-2\right)\right)}{8\,\Gamma(\gamma+1)^2\left(\mathrm{e}^{\frac{(x+y)}{\sqrt{2}}}+1\right)^5}\frac{\tau^{3\gamma}}{\Gamma(3\gamma+1)}$$
$$\vdots \tag{22}$$

Therefore, the m^{th} NDM- solution of Eq. (18) is:

$$\mathcal{v}_i(x,y,\tau)=\sum_{j=0}^{m}\mathcal{v}_j(x,y,\tau)=\mathcal{v}_0(x,y,\tau)+\mathcal{v}_1(x,y,\tau)+\mathcal{v}_2(x,y,\tau)+\cdots+\mathcal{v}_m(x,y,\tau) \tag{23}$$

It is worth mentioning that as $\gamma=1$, the iterations in Eq. (22) are:

$$\mathcal{v}_0=\frac{1}{\mathrm{e}^{(x+y)/\sqrt{2}}+1},\mathcal{v}_1=\frac{\mathrm{e}^{(x+y)/\sqrt{2}}}{2\left(\mathrm{e}^{(x+y)/\sqrt{2}}+1\right)^2}\frac{\tau}{1!},\mathcal{v}_2=\frac{\mathrm{e}^{(x+y)/\sqrt{2}}\left(\mathrm{e}^{(x+y)/\sqrt{2}}-1\right)}{2\left(\mathrm{e}^{(x+y)/\sqrt{2}}+1\right)^3}\frac{\tau^2}{2!},$$
$$\mathcal{v}_3=\frac{e^{(x+y)/\sqrt{2}}\left(-4e^{(x+y)/\sqrt{2}}+e^{\sqrt{2}(x+y)}+1\right)}{8\left(e^{(x+y)/\sqrt{2}}+1\right)^4}\frac{\tau^3}{3!},\cdots \tag{24}$$

The components in (24) can be rewritten in closed form series as the classical Taylor series expansion at $\tau=0$ of the function $\mathcal{v}(x,y,\tau)=1/(1+e^{(x+y-\tau/\sqrt{2})/\sqrt{2}})$ that meets the analytical solution considered by Tchier et al. [40].

For comparison purposes, the fourth approximation is taken into consideration. The known exact solution of the two-dimensional integer order Fisher equation Eq. (18) is $v(x,y,\tau)=\left(1+e^{(x+y-\tau/\sqrt{2})/\sqrt{2}}\right)^{-1}$ [40]. In (Table **1**), the exact solution is compared to the fourth-order term NDM-solution. The results obtained are converging rapidly and achieving a considerably high level of accuracy compared to the numerical results data of Baranwal et al. [39] and Tchier et al. [40]. Furthermore, for smaller time values, the NDM solution is more accurate than for larger values of τ. In Figure 1, the graph of the exact solution at $\gamma=1$ versus the fourth NDM-solution at $\gamma=0.1, 0.8, 1$ is displayed for various values of time. The absolute error is illustrated (Figure **2**), proving the rapid convergence of the method, emphasising that the absolute error accumulates as NDM-solution approaches the space values where $x=y$.

Table 1. Comparison between the NDM-solution and the exact solution (**$x=y=0.5$)***

	$\mathcal{v}_2(x,y,\tau)(\gamma=0.8)$			$\mathcal{v}_4(x,y,\tau)(\gamma=1)$			
τ	**NIM [39]**	**RPSM [40]**	**NTM**	**NTM**	**Exact**	**Absolute Error**	**Absolute Error [40]**
0.01	0.333229	0.333229	0.333229	0.331345	0.331345	1.33227×10^{-15}	7.56291 ´ 10^{-10}
0.05	0.341156	0.341156	0.341153	0.335791	0.335791	4.36773×10^{-12}	9.588 ´ 10^{-8}
0.1	0.349387	0.349387	0.349375	0.34139	0.34139	1.44141×10^{-10}	7.8041 ´ 10^{-7}
0.15	0.356899	0.356899	0.356866	0.347033	0.347033	1.12784×10^{-9}	2.67873 ´ 10^{-6}
0.2	0.364004	0.364004	0.363938	0.352719	0.352719	4.89318×10^{-9}	6.4552 ´ 10^{-6}

Received*: 08-04-2026 -* ***Accepted****: 06-06-2026 -* ***Published****: 11-06-2026*

https://doi.org/10.54361/ajmas.269613

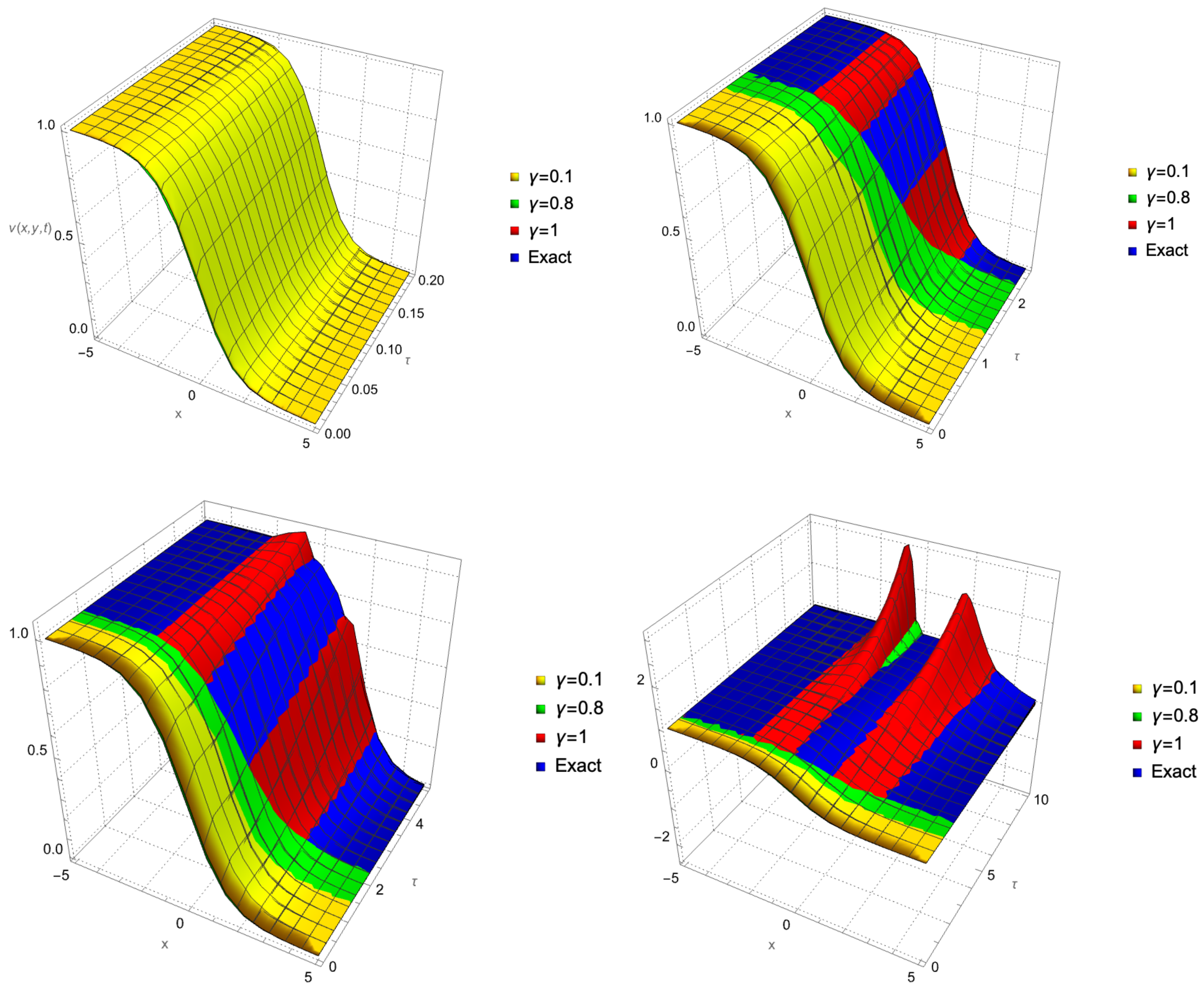

Figure 1. The surface graph of the exact solution (19) when $\gamma = 1$ compared to $v_4(x, y, \tau)$ for $\gamma = 0.1, 0.8, 1$ at $\tau = 0.2, 2.5, 5, 10$ (respectively).

The illustrations in (Figure **1**) indicate that as the time increases, the accuracy of the NDM-solution decreases, so more terms are needed to achieve higher agreement between the exact and the approximate one.
In (Figure **2**) the behavior of the absolute errors is plotted and compared over a range of time. The results obtained signify that the accuracy of the method decreases as the time value increases. However, the technique is promising, and the results have revealed and proved the reliability of solving the multi-dimensional fractional reaction-diffusion equation in Caputo's sense.

Received*: 08-04-2026 -* ***Accepted****: 06-06-2026 -* ***Published****: 11-06-2026*

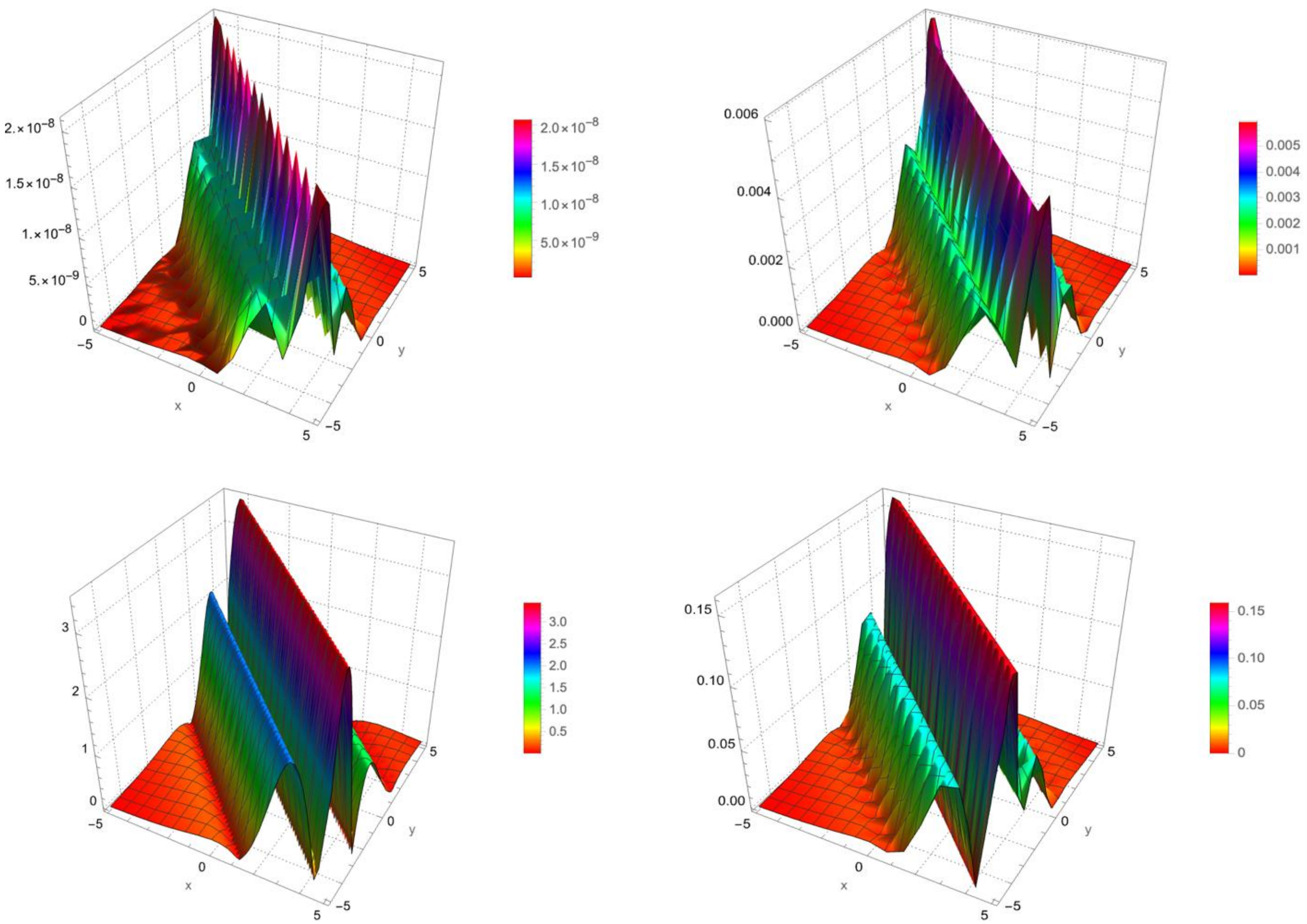

Figure 2. The 3D profile of the absolute error, at $\tau = 0.2, 2.5, 5, 10$ ***(respectively)***

Conclusion

The fractional NDM is successfully employed to numerically approximate the solution of the two-dimensional time-fractional Fisher equation, which has an undeniable role in scientific fields such as chemical kinetics and autocatalytic chemical reactions, etc. The series solution converges rapidly with a highly accurate approximation. Not only did the reliability and the applicability of the method demonstrated via Mathematica software-generated illustrations of the exact solution and numerical one with different fractional orders, but also a confirmation of its validity to be implemented for multi-dimensional fractional problems that appear in different branches of science.

Conflicts of interest

"The authors declare that they have no conflicts of interest.".

References

1. Podlubny I. Fractional Differential Equations. Vol 198. New York: Academic Press; 1999.
2. Hilfer R. Applications of Fractional Calculus in Physics. World Scientific; 2000. doi: 10.1142/3779.
3. El-Horbaty MM, Ahmed F. Numerical solutions of some nonlinear partial differential equations by the decomposition method. J Fac Educ. 2016;6:1–14.
4. Shi XC, Huang LL, Zeng Y. Fast Adomian decomposition method for the Cauchy problem of the time-fractional reaction diffusion equation. Adv Mech Eng. 2016;8(2):1–5. doi: 10.1177/1687814016629898.
5. Pandey P, Kumar S, Gómez-Aguilar JF, B. D. An efficient technique for solving the space-time fractional reaction-diffusion equation in porous media. Chin J Phys. 2020;68:483–492. doi: 10.1016/j.cjph.2020.09.031.

***Received**: 08-04-2026 - **Accepted**: 06-06-2026 - **Published**: 11-06-2026*

6. Alaa A-Q, Zid O, Shawagfeh N. A linearization-based computational algorithm of homotopy analysis method for nonlinear reaction–diffusion systems. Math Comput Simul. 2022;194:505–522. doi: 10.1016/j.matcom.2021.11.027.
7. Ahmed FM, Mansour MA, El-Horbaty M. Solitary wave solution for the time-fractional modified KdV-Burger's equation by using the residual power series method. Afro-Asian J Sci Res. 2025 Dec;3(4):238–243. Accessed January 16, 2026. Available from: https://aajsr.com/index.php/aajsr/article/view/720
8. Sun J-S. Approximate analytic solution of the fractional Fisher's equation via fractional variational iteration method. Adv Mech Eng. 2022;26(3):2699–2705. Available from: https://journals.sagepub.com/doi/pdf/10.1177/1687814016629898
9. Shah R, Khan H, Mustafa S, Kumam P, Arif M. Analytical solutions of fractional-order diffusion equations by natural transform decomposition method. Entropy (Basel). 2019 Jun;21(6):557. doi: 10.3390/E21060557.
10. Singkai CW, Saha UK. A novel numerical approach for solving generalized fractional Fisher's equation. Phys Scr. 2025 Jan;100(2):025241. doi: 10.1088/1402-4896/ada401.
11. Pariyar S, Kafle J. Fractional advection-diffusion equation with variable diffusivity: pollutant effects using Adomian decomposition method. 2025. Available from: http://www.math.nthu.edu.tw/
12. Sinha VK, Maroju P. A new variational iteration method for solving reaction-diffusion equations. Phys Scr. 2025 May;100(6):065233. doi: 10.1088/1402-4896/add8f6.
13. Yadav JU, Singh TR. Alternative variational iteration Elzaki transform method for solving time-fractional generalized Burgers-Fisher equation in porous media flow modeling. Math Methods Appl Sci. 2025 Jun;48(9):9853–9865. doi: 10.1002/mma.10848.
14. Pant R, Arora G, Emadifar H, Hamoud AA. Sumudu residual power series method to solve time-fractional Fisher's equation. Inf Sciences. 2025 Feb;4. doi: 10.1142/S281100722450010X.
15. Bala Sai Sankar B, Karunakar P. Solution of imprecisely defined nonlinear time-fractional Fisher equation using fractional reduced differential transform method. Eng Comput. 2025 May;42(4):1554–1576. doi: 10.1108/EC-09-2024-0834.
16. Pavani K, Raghavendar K, Aruna K. An efficient technique to study time fractional extended Fisher-Kolmogorov equation. Contemp Math. 2024;5(2):1200–1215. doi: 10.37256/cm.5220242433.
17. Onyejekwe ON. Semi-analytical solution of a two-dimensional time-fractional Fisher's equation via the homotopy analysis method. Earthline J Math Sci. 2025 Nov;15(6):1193–1201. doi: 10.34198/ejms.15625.11931201.
18. Alkan A, Bulut H, Çelik E. The novel numerical solutions for time-fractional Fishers equation. Adv Math Phys. 2026 Jan;2026(1):6635126. doi: 10.1155/admp/6635126.
19. Sebaq MS, Qamlo AH, Bahaa GM. Numerical solutions for fractional optimal control problems of coupled diffusion systems via Laplace Adomian Decomposition Method. Boundary Value Probl. 2025 Sep;2025(1):131-. doi: 10.1186/s13661-025-02108-5.
20. Hossain M, Ullah MA. Laplace Adomian decomposition method and Laplace variational iteration method approaches to the nonlinear space-time fractional Fokker-Planck equation. Ric Mat. 2026 Jan:1–37. doi: 10.1007/s11587-025-01050-z.
21. Ullah S, Ali R. Stability analysis of semi-analytical technique for time-fractional Cauchy reaction-diffusion equations. Z Naturforsch A. 2025 Mar;80(3):199–214. doi: 10.1515/zna-2024-0201.
22. Kashyap M, Gupta S, Arora HD, Verma AK. Numerical analysis of nanoparticle diffusion: solving time-fractional Klein–Gordon equations with the Laplace homotopy perturbation method. Stud Syst Decis Control. 2025;235:223–240. doi: 10.1007/978-3-031-84955-8_10.
23. Kapoor M. Implementation of Shehu Adomian Decomposition Method upon Fisher's equation for analytical solution. Discontinuity Nonlinearity Complex. 2025;14(2):337–355. doi: 10.5890/DNC.2025.06.008.
24. Al-deiakeh R, Alhazmi S, Al-Omari S, Al-Smadi M, Momani S. On the Laplace residual series method and its application to time-fractional Fisher's equations. Fractal Fract. 2025 Apr;9(5). doi: 10.3390/fractalfract9050275.
25. Ali KK, Mohamed MS, Maneea M, Botros M. Solutions of nonlinear Murray equation for blood flow in vessels by Laplace-residual power series. Eng Comput. 2025 Nov;42(7):2478–2499. doi: 10.1108/EC-04-2025-0324.
26. Jalili M. A novel homotopy perturbation Sumudu transform method for nonlinear fractional PDEs: applications and comparative analysis. arXiv [Preprint]. 2025. arXiv:2506.20457. doi: 10.48550/ARXIV.2506.20457.
27. Sarwe DU, Raj ASA, Kumar P, Salahshour S. A novel analysis of the fractional Cauchy reaction-diffusion equations. Indian J Phys. 2024 Sep;99(5):1825–1837. doi: 10.1007/s12648-024-03411-0.
28. Loyinmi AC, Akinfe TK, Adedapo Loyinmi CC. Exact solutions to the family of Fisher's reaction-diffusion equation using Elzaki homotopy transformation perturbation method. Eng Rep. 2020;2. doi: 10.1002/eng2.12084.
29. Zidan AM, Khan A, Shah R, Alaoui MK, Weera W. Evaluation of time-fractional Fisher's equations with the help of analytical methods. AIMS Math. 2022;7(10):18746–18766. doi: 10.3934/math.20221031.
30. Hosseini K, Ilie M, Mirzazadeh M, Baleanu D. An analytic study on the approximate solution of a nonlinear time-fractional Cauchy reaction–diffusion equation with the Mittag–Leffler law. Math Methods Appl Sci. 2021 May;44(8):6247–6258. doi: 10.1002/mma.7059.
31. Pandey P, Kumar S, Gómez F. Approximate analytical solution of two-dimensional space-time fractional diffusion equation. Math Methods Appl Sci. 2020 Aug;43(12):7194–7207. doi: 10.1002/mma.6456.
32. Pant R, Arora G, Singh BK, Emadifar H. Numerical solution of two-dimensional fractional differential equations using Laplace transform with residual power series method. Nonlin Eng. 2024 Jan;13(1). doi: 10.1515/nleng-2022-0347.

***Received**: 08-04-2026 - **Accepted**: 06-06-2026 - **Published**: 11-06-2026*

33. Akshey, Singh TR. Approximate analytical solution of the time-fractional reaction-diffusion-convection equation using Aboodh transform iterative method. Iranian J Sci. 2025 Jun;49(3):757–769. doi: 10.1007/s40995-024-01739-0.
34. Rawashdeh MS, Maitama S. Solving coupled system of nonlinear PDE's using the natural decomposition method. Int J Pure Appl Math. 2014;92(5):757–776. doi: 10.12732/ijpam.v92i5.10.
35. Rawashdeh MS. The fractional natural decomposition method: theories and applications. Math Methods Appl Sci. 2017 May;40(7):2362–2376. doi: 10.1002/mma.4144.
36. Bushnaq S, Ali A, Abdullah. Numerical investigation of fractional Fisher partial differential equation via natural transform decomposition method. Partial Differ Equ Appl Math. 2024 Mar;9. doi: 10.1016/j.padiff.2024.100642.
37. Usman M, Khan HU, Khan ZA, Alrabaiah H. Study of nonlinear generalized Fisher equation under fractional fuzzy concept. AIMS Math. 2023;8(7):16479–16493. doi: 10.3934/math.2023842.
38. Veeresha P, Prakasha DG. An efficient technique for two-dimensional fractional order biological population model. Int J Model Simulation Sci Comput. 2020 Feb;11(1). doi: 10.1142/S1793962320500051.
39. Baranwal VK, Pandey RK, Tripathi MP, Singh OP. An analytic algorithm for time fractional nonlinear reaction-diffusion equation based on a new iterative method. Commun Nonlinear Sci Numer Simul. 2012;17(10):3906–3921. doi: 10.1016/j.cnsns.2012.02.015.
40. Tchier F, Inc M, Korpinar ZS, Baleanu D. Solutions of the time fractional reaction-diffusion equations with residual power series method. Adv Mech Eng. 2016 Sep;8(10):1–10. doi: 10.1177/1687814016670867.
41. Belgacem FBM, Silambarasan R. Theory of natural transform. J MESA. 2012;3(1):105–135.
42. Khan ZH, Khan WA. N-Transform - Properties and Applications. NUST J Eng Sci. 2008 Dec;1(1):127–133. doi: 10.24949/NJES.V1I1.69.
43. Kilbas AA, Srivastava HM, Trujillo JJ. Theory and Applications of Fractional Differential Equations. Amsterdam: Elsevier; 2006.
44. Belgacem FBM, Silambarasan R. Advances in the Natural transform. AIP Conf Proc. 2012 Nov;1493(1):106–110. doi: 10.1063/1.4765477.
45. Rawashdeh MS, Al-Jammal H. Theories and applications of the inverse fractional natural transform method. Adv Difference Equ. 2018 Jun;2018(1):222. doi: 10.1186/s13662-018-1673-0.
46. Adomian G. Solution of physical problems by decomposition. Comput Math Appl. 1994;27(9–10):145–154. doi: 10.1016/0898-1221(94)90132-5.

Received*: 08-04-2026 -* ***Accepted****: 06-06-2026 -* ***Published****: 11-06-2026*