*Original article*

# Application of the $(G'/G)$- Expansion Method for The Nonlinear Variant Boussinesq Equation and Drinfeld-Sokolov Equation

**Safaa Abu-Amr[1]*, Ruweydah Tahir[2]**

[1]Department of Mathematics, Faculty of Science, Omar Al-Mokhtar University, El-Baida – Libya
[2]Department of Mathematics, Faculty of Education, Derna University, Derna– Libya
***Email.*** safaa.soliman@omu.edu.ly

**Abstract**
In this paper, $(G'/G)$- The expansion method has been applied to find the traveling wave solutions of the Variant Boussinesq equation and Drinfeld-Sokolov equation. It has been shown that the expansion method saves time and effort in obtaining explicit and accurate solutions, thus providing insights into the dynamics of nonlinear systems. This method has the ability to simplify difficult and complex equations. An effective approach for deriving explicit solutions, offering significant insights into the dynamics of nonlinear systems. Furthermore, it highlights the method's ability to simplify complex equations, establishing it as a valuable tool for both theoretical research and practical applications.
**Keywords.** The $(G'/G)$- Expansion Method, Variant Boussinesq Equation, Drinfeld-Sokolov Equation.

## Introduction

The $(G'/G)$ This method is a systematic analytical approach employed to represent the governing equations of the problem. It relies on representing solutions as a function whose logarithmic derivative satisfies a simple auxiliary equation, thereby reducing complex nonlinear problems to solvable forms. Owing to its clarity and efficiency, this method has been successfully utilized in the analysis of various nonlinear models [1-13].

## Description of the $(G'/G)$- expansion method for NPDEs

In this section, we will explain the basic way of the $(G'/G)$- expansion method discussed in [14].

$$P(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \dots) = 0 \qquad (.21)$$

Where $u = u(x,t)$ is an unknown function, $P$ is a polynomial in $u = u(x,t)$ and its
Various partial derivatives involve the highest order derivatives and nonlinear terms. Below, we outline the key steps of the (G'/G)-expansion method:

**Step 1.** Suppose that

$$u(x,t) = u(\xi), \quad \xi = \xi(x,t) \qquad (.22)$$

The variable associated with a traveling wave (.22) allows us to reduce (.21) to an ODE for $u(\xi) = u(\xi)$ in the form:

$$P(u, u', u'', \dots) = 0 \qquad (.23)$$

Where $' = \frac{d}{d\xi}$

**Step 2.** Assuming the solution described in Eq. (.23) can be represented using a polynomial in (G'/G) as follows:

$$u(\xi) = \sum_{i=0}^{n} a_i \left(\frac{G'}{G}\right)^i \qquad (.24)$$

While $G = G(\xi)$ satisfies the second-order linear differential equation expressed in the form:

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \qquad (.25)$$

Where $a_i (i = 0,1,\dots,n), \lambda$ *and* $\mu$ are constants to be determined later.

**Step 3.** The positive integer "n" can be identified by analyzing the homogeneous balance between the highest derivative term and the nonlinear terms presented in Eq. (.23) as outlined below:

If $D[u(\xi)] = n$, then $D\left[u^r \left(\frac{d^q u}{d\xi^q}\right)^s\right] = nr + s(q+n)$, Here, D represents the degree of the expression.

**Step 4.** We change Eq.(.2 4) into Eq.(.2 3) and apply Eq.(2.5 ), grouping all terms that share the same power of (G'/G), and subsequently setting each coefficient of the resulting polynomial to zero, which produces a system of algebraic equations for $a_i, \lambda, \mu, c$ *and* $k$.

**Step 5.** By solving the algebraic equations using Maple or Mathematica, we determine the values for $a_i, \lambda, \mu, c$ *and* $k$.

**Step 6.** Since the general solutions of Eq. (.25), let's be clear that it's known by the method of substitution. The solution in every respect Eq. (.25) into Eq. (.24), we obtain the traveling wave solutions for the nonlinear PDE (.21).


***Received****: 12-02-2026 -* ***Accepted****: 09-04-2026 -* ***Published****: 15-04-2026*

https://doi.org/10.54361/ajmas.269416

## Application of the method

### *Variant Boussinesq Equation [18]*

emerges as a key framework for capturing the propagation of extended nonlinear waves in dispersive media, integrating both nonlinear and dispersive contributions. Its formulation enables a detailed examination of wave configurations, stability behavior, and interactions within fluid systems, while offering a computational perspective that surpasses the fidelity of standard linear wave approaches.

$$u_t + auu_x + bu^2u_x + cu_{xxx} + du_{xyy} = 0, \quad (3.1.1)$$

$$v_t + (uv)_x + u_{xxx} = 0, \quad (3.1.2)$$

$$\xi = x - ct, \quad (3.1.3)$$

$$-cu' + v' + uu' = 0, \quad (3.1.4)$$

$$-cv' + uv' + u'v + u''' = 0, \quad (3.1.5)$$

$n = 1, m = 2$. Thus, we have

$$u(\xi) = a_0 + a_1\left(\frac{G'}{G}\right), \; a_1 \neq 0, \quad (3.1.6)$$

$$v(\xi) = a_0 + a_1\left(\frac{G'}{G}\right) + a_2\left(\frac{G'}{G}\right)^2, \quad a_2 \neq 0, \quad (.3.17)$$

$$\left(\frac{G'}{G}\right)^0 : Ca_1\mu - a_0a_1\mu - b_1\mu = 0,$$

$$\left(\frac{G'}{G}\right)^1 : Ca_1\lambda - a_0a_1\lambda - a_1^2\mu - b_1\lambda - 2b_2\mu = 0,$$

$$\left(\frac{G'}{G}\right)^2 : -a_1^2\lambda + Ca_1 - a_0a_1 - 2b_2\mu - b_1 = 0,$$

$$\left(\frac{G'}{G}\right)^3 : -a_1^2 - 2b_2 = 0,$$

$$\left(\frac{G'}{G}\right)^0 : -a_1\lambda^2\mu + Cb_1\mu - a_0b_1\mu - a_1b_0\mu - 2a_1\mu^2 = 0, \quad (3.1.8)$$

$$\left(\frac{G'}{G}\right)^1 : -a_1\lambda^3 + Cb_1\lambda + 2Cb_2\mu - a_0b_1\lambda - 2a_0b_2\mu - a_1b_0\lambda - 2a_1b_1\mu - 8a_1\lambda\mu = 0,$$

$$\left(\frac{G'}{G}\right)^2 : 2Cb_2\lambda - 2a_0b_2\lambda - 2a_1b_1\lambda - 3a_1b_2\mu - 7a_1\lambda^2 + Cb_1 - a_0b_1 - a_1b_0 - 8a_1\mu = 0,$$

$$\left(\frac{G'}{G}\right)^3 : -3a_1b_2\lambda + 2Cb_2 - 2a_0b_2 - 2a_1b_1 - 12a_1\lambda = 0,$$

$$\left(\frac{G'}{G}\right)^4 : -3a_1b_2 - 6a_1 = 0,$$

$$C = C, \quad a_0 = a_0, \; a_1 = 0, \quad b_0 = b_0, \quad b_1 = 0, \quad b_2 = 0, \quad (3.1.9)$$

$$C = a_0 - \lambda, \quad a_0 = a_0, \; a_1 = 2, \; b_0 = -2\mu, \quad b_1 = -2\lambda, \; b_2 = -2, \quad (3.1.10)$$

$$C = a_0 + \lambda, \quad a_0 = a_0, \; a_1 = -2, \quad b_0 = -2\mu, \quad b_1 = -2\lambda, \; b_2 = -2, \quad (3.1.11)$$

$$u(\xi) = a_0 + 2\left(\frac{G'}{G}\right), \quad (3.1.12)$$

$$v(\xi) = -2\mu - 2\lambda\left(\frac{G'}{G}\right) - 2\left(\frac{G'}{G}\right)^2, \quad (3.1.13)$$

Where

$$\xi = x - (a_0 + \lambda)t, \quad (3.1.14)$$

If $M > 0$, then

$$u(\xi) = a_0 + 2\left[\frac{1}{2}\sqrt{M}\left(\frac{A\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right) + B\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}{A\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right) + B\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}\right) - \frac{\lambda}{2}\right], \quad (3.1.15)$$

$$u(\xi) = a_0 + \sqrt{M}\left(\frac{A\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right) + B\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}{A\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right) + B\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}\right) - \lambda, \quad (3.1.16)$$

$$v(\xi) = -2\mu - 2\lambda\left[\frac{1}{2}\sqrt{M}\left(\frac{A\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right) + B\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}{A\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right) + B\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}\right) - \frac{\lambda}{2}\right] - 2\left[\frac{1}{2}\sqrt{M}\left(\frac{A\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right) + B\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}{A\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right) + B\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}\right) - \frac{\lambda}{2}\right]^2,$$

(3.1.17)

$$v(\xi) = -2\mu + \frac{\lambda^2}{2} - \frac{M}{2}\left(\frac{A\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right) + B\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}{A\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right) + B\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}\right)^2, \quad (3.1.18)$$

If $M < 0$, then

$$u(\xi) = a_0 + 2\left[\frac{1}{2}\sqrt{-M}\left(\frac{-A\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right) + B\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}{A\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right) + B\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}\right) - \frac{\lambda}{2}\right],$$

$$u(\xi) = a_0 + \sqrt{-M}\left(\frac{-A\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right) + B\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}{A\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right) + B\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}\right) + \lambda, \quad (3.1.19)$$


***Received**: 12-02-2026 - **Accepted**: 09-04-2026 - **Published**: 15-04-2026*

$$v(\xi) = -2\mu - 2\lambda\left[\frac{1}{2}\sqrt{-M}\left(\frac{-A\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right)+B\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}{A\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right)+B\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}\right)-\frac{\lambda}{2}\right] - 2\left[\frac{1}{2}\sqrt{-M}\left(\frac{-A\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right)+B\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}{A\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right)+B\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}\right)-\frac{\lambda}{2}\right]^2,$$

$$v(\xi) = -2\mu + \frac{\lambda^2}{2} - \frac{(-M)}{2}\left(\frac{-A\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right)+B\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}{A\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right)+B\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}\right)^2, \quad (3.1.20)$$

If $M = 0$,

$$u(\xi) = a_0 + 2\left[\frac{B}{B\xi+A} - \frac{\lambda}{2}\right], \quad (3.1.21)$$

$$u(\xi) = a_0 + \frac{2B}{B\xi+A} - \lambda, \quad (3.1.22)$$

$$v(\xi) = -2\mu - 2\lambda\left[\frac{B}{B\xi+A} - \frac{\lambda}{2}\right] - 2\left[\frac{B}{B\xi+A} - \frac{\lambda}{2}\right]^2, \quad (3.1.23)$$

$$v(\xi) = -2\mu + \frac{\lambda^2}{2} - 2\left(\frac{B}{B\xi+A}\right)^2, \quad (3.1.24)$$

If $M > 0$,

$$u(\xi) = a_0 - 2\left[\frac{1}{2}\sqrt{M}\left(\frac{A\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right)+B\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}{A\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right)+B\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}\right)-\frac{\lambda}{2}\right], \quad (3.1.25)$$

$$v(\xi) = -2\mu - 2\lambda\left[\frac{1}{2}\sqrt{M}\left(\frac{A\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right)+B\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}{A\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right)+B\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}\right)-\frac{\lambda}{2}\right] - 2\left[\frac{1}{2}\sqrt{M}\left(\frac{A\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right)+B\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}{A\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right)+B\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}\right)-\frac{\lambda}{2}\right]^2$$

(3.1.26)

If $M = 0$,

$$u(\xi) = a_0 - 2\left[\frac{B}{B\xi+A} - \frac{\lambda}{2}\right], \quad (3.1.27)$$

$$u(\xi) = a_0 - \frac{2B}{B\xi+A} + \lambda, \quad (3.1.28)$$

$$v(\xi) = -2\mu - 2\lambda\left[\frac{B}{B\xi+A} - \frac{\lambda}{2}\right] - 2\left[\frac{B}{B\xi+A} - \frac{\lambda}{2}\right]^2, \quad (3.1.29)$$

$$v(\xi) = -2\mu + \frac{\lambda^2}{2} - 2\left(\frac{B}{B\xi+A}\right)^2, \quad (3.1.30)$$

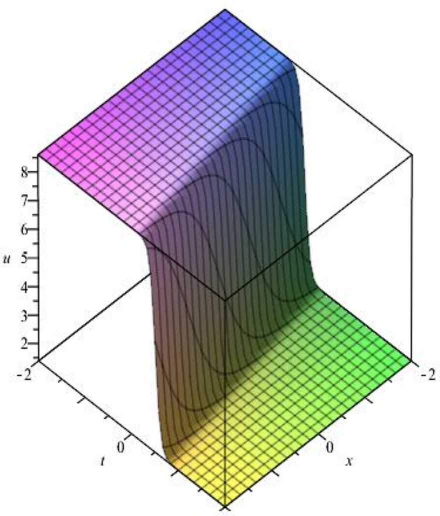


***Fig.3.1 plot(3.1.17)***
$a_0 = 2, \lambda = -3, \mu = -1, A = 3, B = 4, c = 5$

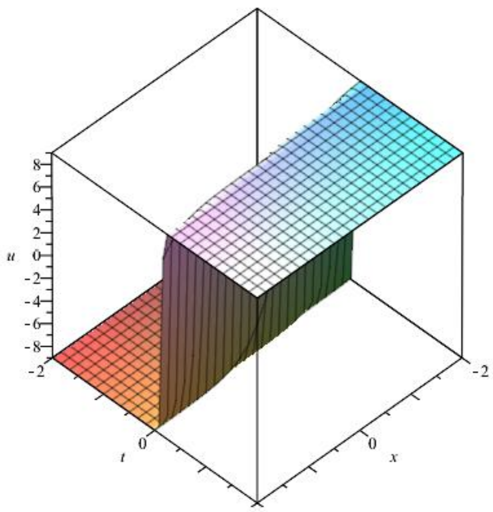


***Fig. 3.1 plot (3.1.2)***
$a_0 = 2, \lambda = -3, \mu = -1, A = 3, B = 4, c = 5$


***Received**: 12-02-2026 - **Accepted**: 09-04-2026 - **Published**: 15-04-2026*

https://doi.org/10.54361/ajmas.269416

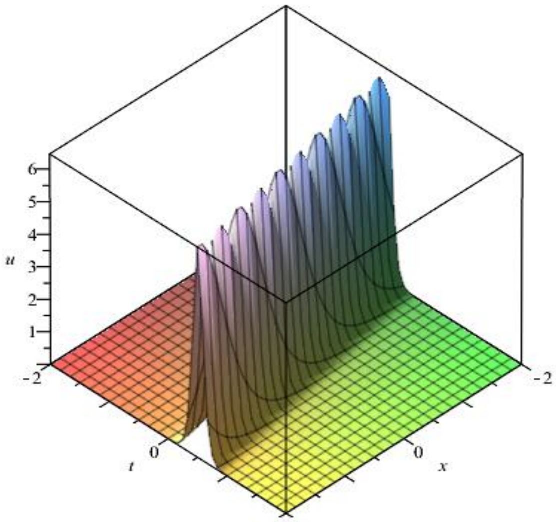


***Fig.3.1 plot (3.1.18)***

$a_0 = 2, \lambda = -3, \mu = -1, A = 3, B = 4, c = 5.$

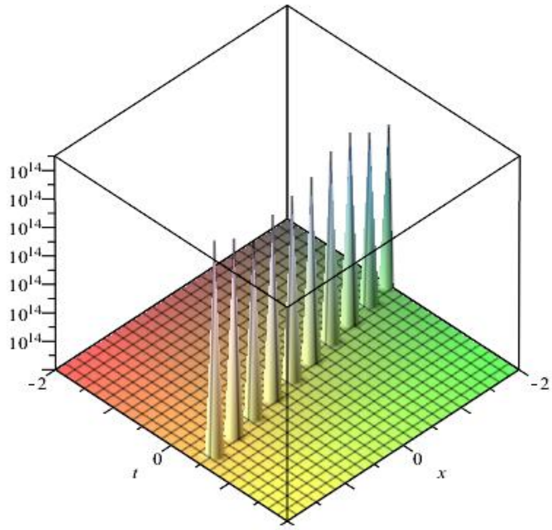


***Fig.3.1 plot (3.1.22)***

$a_0 = 2, \lambda = -3, \mu = -1, A = 3, B = 4, c = 5$

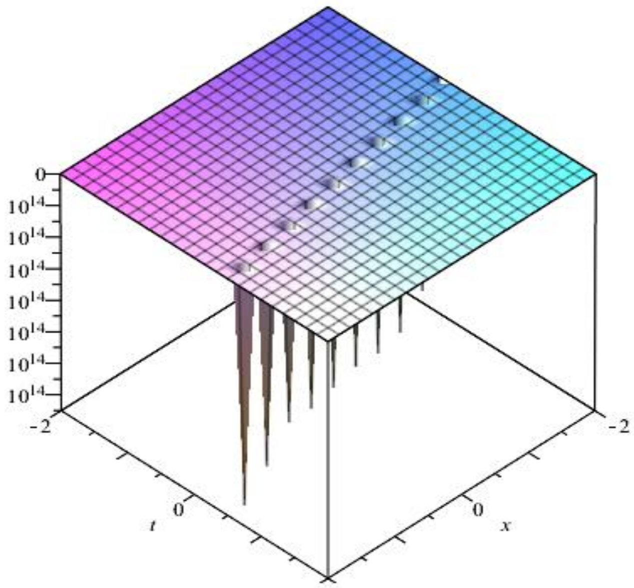


***Fig 3.1 plot (3.1.30)***

$a_0 = 2, \lambda = -3, \mu = -1, A = 3, B = 4, c = 5.$

***The Drinfeld-Sokolov equation [19]***

The Drinfeld–Sokolov equation plays a crucial role as it provides a geometric and algebraic framework for integrable nonlinear systems, rooted in its deep connection with Lie algebras and Hamiltonian reduction. From a physical perspective, it gives rise to fundamental integrable models like the KdV and mKdV equations, which describe nonlinear dispersive wave behavior in fluids and plasmas. In essence, it serves as a pivotal element in bridging geometric structures with physically significant nonlinear wave equations.

$$u_t + (v^2)_x = 0, \qquad (3.2.1)$$

$$v_t + av_{xxx} - 3bu_xv - 3kuv' = 0, \qquad (3.2.2)$$

$$\xi = x - ct, \qquad (3.2.3)$$

$$-cu' + (v^2)' = 0, \qquad (3.2.3)$$

$$cv' + av''' - 3bu'v - 3kuv' = 0, \qquad (3.2.4)$$

$$cu + v^2 + k_1 = 0, \qquad (3.2.5)$$

$$c^2v + acv'' - (2b + k)v^3 + k_2 = 0, \qquad (3.2.6)$$

$m$=1. Thus, we have

$$v(\xi) = a_0 + a_1\left(\frac{G'}{G}\right), \quad a_1 \neq 0, \qquad (3.2.7)$$

$$\left(\frac{G'}{G}\right)^0 : aa_1c\lambda\mu - 2a_0^3b - a_0^3k + a_0c^2 = 0,$$


***Received**: 12-02-2026 - **Accepted**: 09-04-2026 - **Published**: 15-04-2026*

https://doi.org/10.54361/ajmas.269416

$$\left(\frac{G\prime}{G}\right)^1: aa_1c\lambda^2 + 2aa_1c\mu - 6a_0^2a_1b - 3a_0^2a_1k + a_1c^2 = 0,$$

$$\left(\frac{G\prime}{G}\right)^2: 3aa_1c\lambda - 6a_0a_1^2b - 3a_0a_1^2k = 0, \tag{3.2.8}$$

$$\left(\frac{G\prime}{G}\right)^3: -2a_1^3b - a_1^3k + 2aa_1c = 0, \tag{3.2.9}$$

$$a_0 = \pm\frac{a\lambda}{2}\sqrt{\frac{\lambda^2-4\mu}{2b+k}} \quad a_1 = \pm\left(\sqrt{\frac{\lambda^2-4\mu}{2b+k}}\right)a, \quad c = \frac{1}{2}a(\lambda^2 - 4\mu) \tag{3.2.10}$$

$$a_0 = \frac{1}{2\sqrt{2b+k}}\mathrm{c}, \quad a_1 = 0, \mathrm{c} = \mathrm{c}, \tag{3.2.11}$$

$$v(\xi) = \pm\frac{a\lambda}{2}\sqrt{\frac{\lambda^2-4\mu}{2b+k}} \pm \left(\sqrt{\frac{\lambda^2-4\mu}{2b+k}}\right)a\left(\frac{G'}{G}\right), \tag{3.2.12}$$

If $M > 0$,

$$v_1(\xi) = \frac{a\sqrt{M}}{2}\sqrt{\frac{\lambda^2-4\mu}{2b+k}}\left(\frac{A\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right)+B\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}{A\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right)+B\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}\right), \tag{3.2.13}$$

$$v_2(\xi) = -\frac{a\sqrt{M}}{2}\sqrt{\frac{\lambda^2-4\mu}{2b+k}}\left(\frac{A\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right)+B\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}{A\sinh\left(\frac{1}{2}\sqrt{M}\,\xi\right)+B\cosh\left(\frac{1}{2}\sqrt{M}\,\xi\right)}\right), \tag{3.2.14}$$

If $M < 0$,

$$v(\xi) = \pm\frac{a\lambda}{2}\sqrt{\frac{\lambda^2-4\mu}{2b+k}} \pm a\left(\sqrt{\frac{\lambda^2-4\mu}{2b+k}}\right)\left[\frac{1}{2}\sqrt{-M}\left(\frac{A\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right)+B\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}{A\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right)+B\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}\right) - \frac{\lambda}{2}\right], \tag{3.2.15}$$

$$v_1(\xi) = \frac{a\sqrt{-M}}{2}\sqrt{\frac{\lambda^2-4\mu}{2b+k}}\left(\frac{-A\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right)+B\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}{A\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right)+B\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}\right), \tag{3.2.16}$$

$$v_2(\xi) = -\frac{a\sqrt{-M}}{2}\sqrt{\frac{\lambda^2-4\mu}{2b+k}}\left(\frac{-A\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right)+B\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}{A\sin\left(\frac{1}{2}\sqrt{-M}\,\xi\right)+B\cos\left(\frac{1}{2}\sqrt{-M}\,\xi\right)}\right), \tag{3.2. 17}$$

If $M = 0$ ,

$$v_1(\xi) = \frac{a\lambda}{2}\sqrt{\frac{\lambda^2-4\mu}{2b+k}} + a\left(\sqrt{\frac{\lambda^2-4\mu}{2b+k}}\right)\left[\frac{B}{B\xi+A} - \frac{\lambda}{2}\right], \tag{3.2.18}$$

$$v_1(\xi) = \frac{aB}{B\xi+A}\sqrt{\frac{\lambda^2-4\mu}{2b+k}}, \tag{3.2.19}$$

$$v_2(\xi) = -\frac{a\lambda}{2}\sqrt{\frac{\lambda^2-4\mu}{2b+k}} - a\left(\sqrt{\frac{\lambda^2-4\mu}{2b+k}}\right)\left[\frac{B}{B\xi+A} - \frac{\lambda}{2}\right], \tag{3.2.20}$$

$$v_2 = -\frac{aB}{B\xi+A}\sqrt{\frac{\lambda^2-4\mu}{2b+k}}, \tag{3.2.21}$$

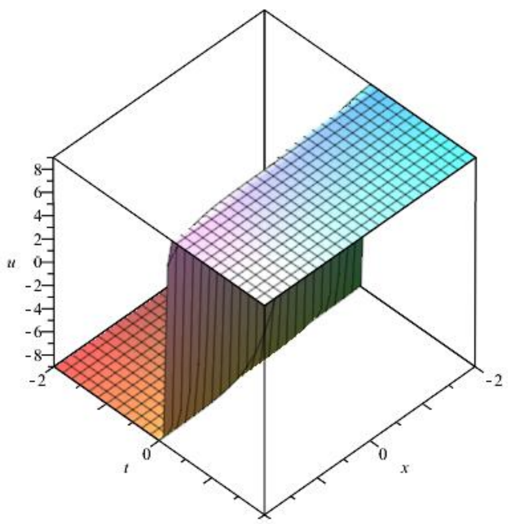


***Fig. 3.2 plot (3.2.22)*** $a = 3, b = 1, k = 2, \lambda = 4, \mu = 1, A = 0.4, B = 0.6, c = 18$

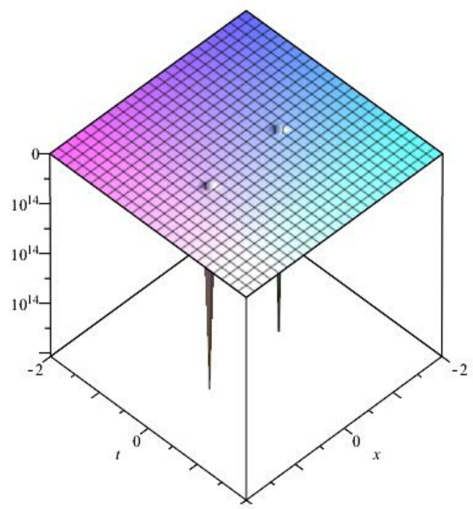


***Fig. 3.2plot(3.2.15)*** $a = 3, b = 1, k = 2, \lambda = 4, \mu = 1, A = 0.4, B = 0.6, c = 18$


**Received**: 12-02-2026 - **Accepted**: 09-04-2026 - **Published**: 15-04-2026

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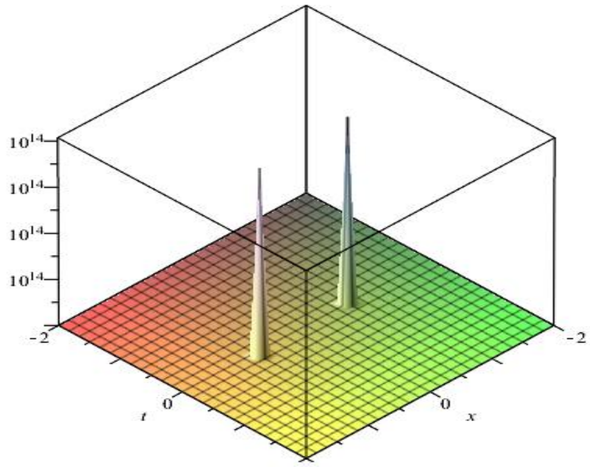


***Fig. 3.2 plot (3.2.20)***

$a = 3, b = 1, k = 2, \lambda = 4, \mu = 1, A = 0.4, B = 0.6, c = 18$

## Conclusion

In this work, we present new applications of the (G′⁄G)- expansion method to construct a series of some new traveling wave solutions for some nonlinear partial differential equations, via the Variant Boussinesq equation and Drinfehd-Sokolov equation.

The performance of this method is found to be effective, powerful and reliable for solving the NPDEs. This method has the advantages of being direct and concise. We also see that this method can be applied widely to many other NPDEs in the mathematical physics and this will be done in a future work.

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***Received****: 12-02-2026 -* ***Accepted****: 09-04-2026 -* ***Published****: 15-04-2026*