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Original Article

Short and Long Memory Models in Modeling Libyan GDP: An Applied Study using ARIMA and ARFIMA

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Abstract
This paper aims to evaluate the ability of time series models in modeling Libyan GDP during the period 1960–2024. The analysis focuses on examining long memory properties and comparing the performance of the Autoregressive Integrated Moving Average (ARIMA) model with the Autoregressive Fractionally Integrated Moving Average (ARFIMA) model. The results showed a general trend and long memory in the series with a fractional difference coefficient d = 0.495795. Unit root tests confirmed the non-stationarity of the original series versus the stationarity of the (fractional/first) difference series. Based on information criteria and forecast measures (AIC, SIC, and RMSE), the ARFIMA (0,0.495795,0) model outperforms traditional ARIMA models in characterizing the dynamics of Libyan GDP. These results provide a practical framework for improving the modeling of economic indicators and formulating economic policies in Libya.
Keywords. ARIMA Model, ARFIMA Model, Fractional Differencing, Short and Long Memory, GDP.

Introduction

Gross Domestic Product (GDP) is one of the most widely used macroeconomic indicators for assessing economic activity and guiding economic policy. Accurate modelling and forecasting of GDP provide policymakers with forward-looking into growth dynamics and recession risks in productive sectors. In Libya's case, the GDP data is characterized by significant changes due to its economy's dependence on oil revenues and its frequent exposure to structural and political shocks [1,2]. These features require the use of quantitative models capable of accurately representing the complex dynamics of Libyan economic data. However, the effectiveness of modeling and forecasting generally depends on choosing a statistical model that accurately reflects the features of the time series data [3].

Linear time series models are among the most effective tools for analyzing and modelling economic phenomena. One of the most common linear models is the ARIMA model, suggested by [4]. This model is based on the assumption that the data can be stationary using the differencing operator (d). Moreover, this model is expressed as a linear mixture of previous values (autoregressive, AR) and previous error terms (moving average, MA) [4]. The ARIMA models have proven effective in characterizing short-memory processes where autocorrelation decreases rapidly with increasing time lag [5].

However, many economic data, including GDP, may exhibit long memory (or long-range depending) properties, where autocorrelation slowly declines but remains statistically significant over long periods [6,7]. Moreover, in such cases, the ability of the ARIMA models to accurately represent data may be limited, and this is where the ARFIMA model proposed by [6,7] becomes necessary. The ARFIMA model is a generalization of the ARIMA model, allowing the order of differencing operator (d) to be a fractional rather than a non-negative integer. Several experimental studies have shown that ARFIMA models provide greater flexibility for analyzing and modeling time series data, and that they can improve forecasting accuracy in cases where the data exhibit properties of fractional stationary and long memory [8,9].

Comparing ARIMA and ARFIMA models is a key practical aspect of time series analysis. Moreover, this comparison is particularly important in the Libyan context, where studies comparing these models remain limited. This comparison helps determine whether changes in Libyan GDP reflect short memory, which can be represented by the ARIMA model, or whether there is persistence of economic shocks, which necessitates the use of the ARFIMA model.

This study aims to examine the presence of long-memory property in Libyan GDP data and compare the forecasting performance of ARIMA and ARFIMA models. The selection of the most suitable model was based on the use of several statistical indicators, such as information criteria (AIC and SIC) and forecast error measures (RMSE and MAE). The study's findings contribute to the applied statistical studies by testing the efficiency of time series models in developing economies described by cyclical instability, such as the Libyan economy. Furthermore, it provides a time series model that can be used to improve the predictive power of future economic indicators.

Theoretical Framework

In macroeconomic analysis, GDP defines as the aggregate value of goods and services produced by economy over a given period (quarterly or annually) and are commonly used to assess economic performance and guide policy decision [2]. Its importance in empirical studies stems from its ability to capture overall economic dynamics, making it a key variable in modelling and forecasting applications [10].

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Time series models are powerful statistical instruments used in data analysis and forecasting, due to their ability to capture trend, seasonal, and cyclical patterns in historical data. Among these models, the ARIMA and the ARFIMA models stand out as popular and effective instruments for time series modelling. Furthermore, the assumptions on which these models are based include stationarity, linearity, and homogeneity of error variance [11,5].

The ARIMA Model

The ARIMA(p,d,q) model developed by [4] in the 1970s is among the most popular linear time series forecasting models. This model is denoted by the formula

$$\varphi_p(B)\,\nabla^d Y_t = \theta_q(B)\varepsilon_t, \qquad (1)$$

Where Y_t denotes the observed time series at time t, p and q represent the orders of the AR and MA components, respectively; and d indicates the degree of differencing required to achieve stationarity. The error term ε_t is assumed to have zero mean and constant positive variance. The operators $\varphi_p(B)$ and $\theta_q(B)$ denote the AR(p) and MA(q) polynomials, respectively. The differencing operator is defined as $\nabla^d = (1-B)^d$, where B is the backshift such that $B\,Y_t = Y_{t-1}$. Here, weakly stationary means that the mean and the variance of the data are constant over time. The strength of the ARIMA model lies in its ability to capture short-term patterns in the data through its limited autocorrelation coefficients. However, one of its main limitations is its assumption that the differencing operation ∇^d is applied with the non-negative integer degree of d, $d \in Z^+$, which means that the model cannot represent long memory in the data [12].

Short and Long Memory Concept in Time Series

ARIMA models depend on the assumption of short memory (short-range dependence), where autocorrelations decline rapidly and exponentially toward zero. In contrast, the concept of long memory (long-range dependence) in the data of time series refers to the presence of autocorrelations that persist for long periods and decrease slowly and hyperbolically towards zero [13,8]. However, this behaviour has attracted attention since the early 1980s with the work of [6,7], who first introduced the fractional difference model (ARFIMA) as a framework to represent this phenomenon. Since then, the term long memory has become an important concept in time series analysis for describing processes that exhibit a long memory property that cannot be explained using traditional ARIMA models. The fractionally differenced process ARFIMA(0,d,0) can be defined by

$$(1-B)^d\,Y_t = \varepsilon_t, \qquad -0.5 < d < 0.5, \qquad (2)$$

where ε_t is a white noise series? Nevertheless, the degree of long memory is typically estimated using the fractional difference of the integration parameter d. (Table 1) shows the theoretical meaning of the difference parameter (d).

Table 1. The theoretical meaning of d.

Range of d	Statistical theoretical meaning
-0.5 < d < 0	Stationary, invertible, anti-persistent
d = 0	short-memory stationary process (ARMA)
0 < d < 0.5	stationary process with long memory
0.5 < d < 1	Non-stationary but mean-reverting process
d = 1	*I(1)* Unit root process
d > 1	Non-stationary and non-mean-reverting (possibly explosive)

Sources: [6,7,13,8,14].

The presence of long memory in time series data means that shocks have effects that persist for long periods of time, which sometimes makes standard models such as ARIMA insufficient for representing actual behavior [6,8].

The ARFIMA model

The ARFIMA model introduced by [6,7] is a natural extension of the ARIMA model to include cases of fractional integration. The ARFIMA model is denoted by (p,d,q), and the general form of the model is written as follows:

$$\varphi_p(B)(1-B)^d\,Y_t = \theta_q(B)\varepsilon_t, \qquad d \in (-0.5, 0.5) \qquad (3)$$

Fractional differencing $(1-B)^d$ is defined according to a series with fractional binomial coefficients (the binomial series) as follows

$$(1-B)^d = \sum_{k=0}^{\infty} w_k\,B^k, \qquad (4)$$
$$w_k = (-1)^k \binom{d}{k} \text{ and } \binom{d}{k} = \frac{d(d-1)\ldots(d-k+1)}{k!} = \frac{\Gamma(d+1)}{\Gamma(k+1)\,\Gamma(d-k+1)},$$

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Where $\Gamma(.)$ is the gamma function and w_k is calculated as follows

$$w_0 = 1, \qquad w_k = -\frac{d-k+1}{k} w_{k-1}, \qquad k \geq 1$$

For a positive integer d, the model reduces to the traditional ARIMA model (integration of order d). For fractional $d \notin Z$, the model exhibits long memory when 0<d<0.5. Therefore, [8] highlighted the practical importance of this model in time series analysis, particularly for series characterized by slow-decreasing autocorrelation. The ARFIMA model is a suitable framework for analyzing macroeconomic series, including GDP, due to its ability to capture long-term dynamics more accurately than its counterpart, ARIMA.

Literature Review

The literature has included numerous studies on GDP modelling and forecasting using various time series models. Some of these studies focused on the application of the ARIMA and ARFIMA models to evaluate their effectiveness in analyzing GDP dynamics and providing accurate forecasts.

[15] Demonstrated that using models like ARFIMA in GDP analysis leads to a better representation of business cycle features compared to traditional models. The results of the study indicate that the features of the business cycle clearly depend on the degree of integration (d) and on short-term components such as AR and MA. By applying the ARFIMA model to real GDP data in the US and UK, it was shown that the data can be represented with degrees of integration greater than 1 and less than 2, reflecting long-term memory and continuity in shocks. Simulations show that ARFIMA models outperform ARIMA models in their ability to describe business cycle characteristics more accurately

[16] Investigated the long memory in the UK's real GDP from 1851 to 2013 using a multiple ARFIMA-FIGARCH model with explanatory variables. The outcome found that the series is non-stationary, and the value of the fractional coefficient d indicates a very large fractional integral (approximately 1.37), suggesting that shocks may be quasi-permanent.

[17] Applied ARIMA models to model Kenya's GDP for the annual period from 1960 to 2012. The results showed that the ARIMA (2,2,2) model performed best, and its forecasts were adequate and effective.

[18] Used the ARIMA model to model China's GDP from 1978 to 2022. The results show that the ARIMA (0, 2, 0) model has high predictive accuracy. In the study of [19], the ARFIMA and ARIMA models were compared for modelling and forecasting Shanghai's GDP. The study revealed that the ARFIMA model outperformed the ARIMA model in terms of AIC value, while the ARIMA model had some advantages in terms of RMSE.

In general, previous studies show that ARIMA models are widely used in GDP modeling and forecasting, with varying results depending on the economic context and data characteristics. However, there is a clear gap in the literature regarding comprehensive comparisons between ARIMA and ARFIMA models, particularly regarding ARFIMA's ability to capture the long memory in GDP data. Consequently, this paper aims to fill this gap by providing a detailed comparative analysis of the performance of the two models in modeling Libyan GDP.

Methodology

Data Description

The data used in this paper is Libya's Gross Domestic Product (GDP) in local currency (Libyan Dinar) at constant prices, taken from the World Bank Open Data (GDP, constant LCU in billions). The data represent annual GDP values covering the period from 1960 to 2024.

Stationery and Unit Root Tests

Stationarity is a fundamental requirement in time series modeling, where the statistical properties of the series, such as the mean and variance, are assumed to remain constant over time. To ensure this condition is met, the presence of a unit root in the time series is typically checked. In this paper, stationarity was checked using the two most common tests: the augmented Dickey-Fuller (ADF) test [20] and the Phillips-Perron (PP) test [21]. If Stationary is not present, the differencing operator is applied to transform the series into a stationary series.

Long-Memory Confirmation

To detect the potential for long memory in the GDP series, a graphical method was used by examining the behaviour of the autocorrelation function (ACF). A slight decrease in correlation coefficients over time is a visual indicator of fractional integration and long-memory characteristics.

Fractional Differencing Estimation

In this paper, the fractional differencing coefficient d was estimated using the maximum likelihood method (MLE) by the EViews software.

Model Identification

To determine the initial orders of the autoregressive (AR) components p and moving averages (MA) components q in ARIMA/ARFIMA models, the autocorrelation functions (ACF) plots and the partial autocorrelation (PACF) plots were used.

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Estimation
Maximum likelihood estimation (MLE) is used in estimating ARIMA and ARFIMA models, as it is one of the most efficient estimation methods, giving consistent and effective estimators under the assumption that the residues follow a normal distribution.

Estimating the ARFIMA Model
Due to the nature of ARFIMA models, which include the fractional difference coefficient d, the estimation process is more complex than in ARIMA [6,7,22,23]. However, this paper uses two methods: Two-stage estimation and one-stage estimation.
Two-stage estimation: This method is performed as follows:
Estimate the value of the fractional difference d using MLE in the ARFIMA (p,d,q) model (i.e., ARFIMA($0,d,0$)).
Construct the series with fractional differences based on the estimated value of d.
Examine the new series graphically and use it to determine the approximate orders of AR and MA (i.e., using ACF and PACF plots).
Estimate the final ARFIMA model using the chosen orders.
One-stage estimation: In this method, the fractional difference coefficient d is estimated simultaneously with the AR coefficients and the MA coefficients in a single estimation process.

Choosing the Best Model
In time series analysis, selecting the best model is crucial for ensuring forecast accuracy. This process depends on informational criteria that provide a quantitative tool for comparing different models. The most important of these criteria are Akaike Information Criterion, AIC [24], and Schwartz Information Criterion, SIC [25].

$$AIC = -2l/T + 2k/T, \qquad (5)$$

$$SIC = -2l/T + (klog\,T)/T, \qquad (6)$$

Where l is the maximized value of the likelihood function of the model, k is the number of parameters, and T is the sample size. The best model is determined by the lowest value of AIC and SIC.

Diagnostic Checking
In this stage, the validity of the chosen model is evaluated by analyzing the model's residuals to ensure they conform to the model's assumptions. The analysis includes the following tests:
To confirm the absence of autocorrelation of the residues, the ACF and the PACF plots are examined. In addition, the Ljung-Box test [26] is carried out to test the null hypothesis H_0: no autocorrelation, versus the alternative H_1: autocorrelation exists. The test statistic is calculated as follows:

$$Q(m) = T(T+2)\sum_{k=1}^{m}\frac{\hat{\rho}_k^2}{T-k}, \qquad (7)$$

Where T is the sample size, $\hat{\rho}_k$ is the estimated autocorrelation coefficient at the lag k, and m is the number of selected lags. If the p-value is less than the significance level α, the null hypothesis is rejected.
Normality tests: to test the normality of the residues, graphical and statistical methods were used, including the histogram and the Jarque-Bera (JB) test[27]; thus, the null hypothesis of this test (H_0: the data are normal) is rejected if the p-value is less than the significance level α.
Heteroskedasticity tests: for testing heteroskedasticity of variance, the ACF and the Ljung-Box statistic for the squared residual series are examined for evidence of autocorrelation. Also, the ARCH test [28] has been used; the test is performed by estimating the regression of the squared residuals and then using the F-statistic and the TR^2, where R^2 the coefficient of determination and T is the sample size. If the null hypothesis H_0: there are no ARCH effect is rejected, indicating heteroskedasticity.

Forecasting and Comparison
The forecasting phase represents the final stage in evaluating the performance of the models and their use to obtain future forecasts for the time series. The accuracy of these forecasts is evaluated through quantitative criteria such as RMSE and MAE, where lower values indicate better model performance.

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{y}_t - y_t)^2}\ , \quad \text{and} \qquad (8)$$

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|\hat{y}_t - y_t|, \qquad (9)$$

Where y_t denotes the actual value, $\hat{y}_t$ denotes the forecast value, and T denotes the sample size.

Results and Discussion
Figure 1 shows the Libyan GDP series and its correlogram from 1960 to 2024. Visual inspection of the plots indicates the presence of a trend in the series. Moreover, the ACF plot shows that the correlation coefficients lie outside the confidence bounds and persist over long periods of time, and decrease slowly with increasing time lag. These features suggest that the series is nonstationary and exhibits long-range memory behaviour. Therefore, the fractional differencing coefficient d is estimated using the ML method, and the results of this estimation are shown in (Table 2).

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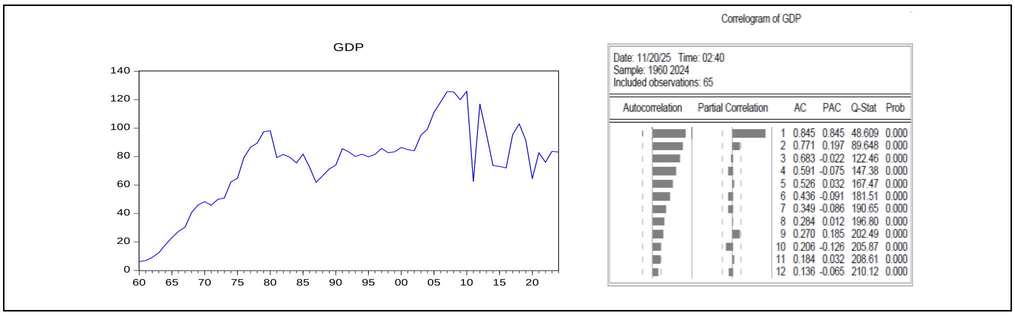

Figure 1. Plots of the Libyan GDP series and its correlogram from 1960 to 2024.

Table 2 illustrates that the estimation value of d is 0.495795 ($0 < d < 0.5$), and it is statistically significant (p-value = 0.000), indicating the presence of long memory. Subsequently, the fractionally differenced GDP series (*FDGDP*) was constructed as follows:

$$FDGDP = (1 - B)^{0.495795} gdp_t$$

where B is the backshift operator. This series represents the GDP data after applying fractional differencing (d = 0.495795) to capture long memory properties. Additionally, the first differenced GDP series (*DGDP*) was also calculated as follows:

$$DGDP = gdp_t - gdp_{t-1}$$

Here, no mathematical transformations such as natural logarithms were applied to the GDP series before the differences were taken. (Figure 2) displays the fractional difference (FDGDP) series, the first difference (*DGDP*) series and their correlogram.

Table 2. Estimation of the Fractional Differencing Coefficient d.

Par.	Coefficient	Std. Error	t-Statistic	Prob.
d	0.495795	0.008179	60.61778	0.0000

From Figure 2, we observe that the (fractional/first) difference series remains stable over time, without any trend. We also note that the ACF and PACF plots for *FDGDP* and *DGDP* decrease rapidly after lag 2 for *FDGDP* and after lag 1 for *DGDP*, indicating that both difference series are stationary.

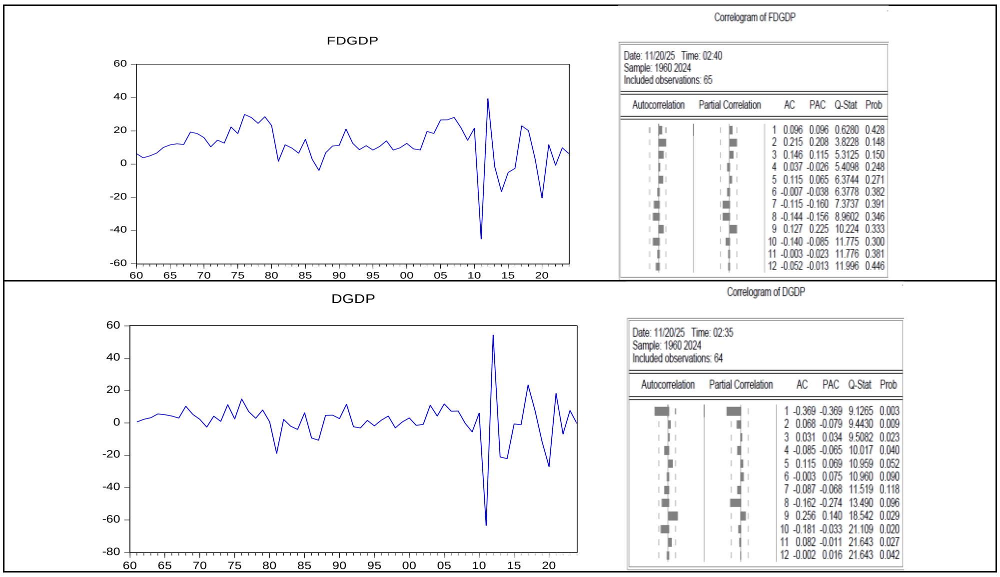

Figure 2. Plots of FDGDP, DGDPand their correlogram.

Table 3 gives the descriptive statistics for the study series; we can note that all series *GDP*, *FDGDP*, and *DGDP* have negative skewness and positive Kurtosis. The JB statistics accepted the null hypothesis of normality for GDP and rejected the null hypothesis of normality for *FDGDP* and *DGDP*. However, to confirm stationarity, the ADF and PP tests with intercept and trend, intercept only, and none were carried out on the GDP and difference series; the findings are shown in (Table 4).

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Table 3. Descriptive statistics of the series data.

Descriptive Statistics	GDP	FDGDP	DGDP
Mean	74.328	11.377	1.201
Maximum	125.947	39.428	54.308
Minimum	6.314	-45.089	-63.399
Std. Dev.	28.917	12.649	13.670
Skewness	-0.614	-1.471	-0.967
Kurtosis	3.190	8.216	12.555
Jarque-Bera	4.184	97.126	253.429
Probability	0.123	0.000*	0.000*
Observations	65	65	64

* *Rejection at 1% significance level.*

The unit root test results in (Table 4) confirm that the null hypothesis is accepted and the Libyan GDP series is non-stationary and has a unit root, as the *p*-values for both ADF and PP tests were greater than the significance level of 0.05. Conversely, the null hypothesis was rejected, and the two series, *FDGDP* and *DGDP,* are stationary, as the *p*-values were less than the significance level of 0.05. Following this, ARIMA and ARFIMA models were estimated using the MLE method at different orders with $p \leq 2$ and $q \leq 2$. Comparisons between these models were performed using AIC and SIC to obtain the optimal model.

Table 4. Results of unit root tests.

Augmented Dickey-Fuller (ADF) Test							
Series	None		Intercept		Intercept and linear trend		Decision
	Test- stat	*p*-value	Test- stat	*p*-value	Test- stat	*p*-value	
GDP	0.1067	0.7129	-2.5606	0.1066	-2.8997	0.1696	Non-stationary
FDGDP	-2.4586	0.0146	-7.1486	0.0000	-7.4101	0.0000	Stationary
DGDP	-11.4544	0.0000	-11.5031	0.0000	-11.6903	0.0000	Stationary
Phillips-Perron (PP) Test							
GDP	-0.0557	0.6604	-2.7222	0.0758	-2.6232	0.2718	Non-stationary
FDGDP	-4.9180	0.0000	-7.3149	0.0000	-7.4749	0.0000	Stationary
DGDP	-11.4544	0.0000	-11.5031	0.0000	-12.1623	0.0000	Stationary

The comparison based on AIC and SIC is presented in (Table 5). In the ARIMA models, the results indicated that the best model with the lowest values in AIC and SIC was the ARIMA (1,1,0) model (AIC=7.9877, and SIC=8.0551). In the ARFIMA models under two-stage estimation, where d = 0.495795, the results indicated that the best model with the lowest values in AIC and SIC was the ARFIMA (0,0.495795,0) model (AIC=7.9284, and SIC=7.9619).

Table 5. Comparison of ARIMA\ ARFIMA models.

Information Criteria	**ARIMA (*p,d,q*)**								
	(0,1,0)	(1,1,0)	(0,1,1)	(1,1,1)	(2,1,0)	(0,1,2)	(2,1,1)	(1,1,2)	(2,1,2)
AIC	8.0838	7.9877	7.9934	8.0160	8.0152	8.0147	8.0192	8.0449	8.0503
SIC	8.1175	8.0551	8.0608	8.1172	8.1164	8.1159	8.1541	8.1799	8.2189
Information Criteria	ARFIMA (*p,d,q*), d = 0.495795\ Two-stage estimation								
	(0,d,0)	(1,d,0)	(0,d,1)	(1,d,1)	(2,d,0)	(0,d,2)	(2,d,1)	(1,d,2)	(2,d,2)
AIC	7.9284	8.2701	8.4019	8.0221	8.1085	8.2758	8.0440	8.0458	8.0741
SIC	7.9619	8.3370	8.4689	8.1225	8.2089	8.3762	8.1778	8.1796	8.2413
Information Criteria	ARFIMA (*p,d,q*)\ One-stage estimation								
	(0,d,0)	(1,d,0)	(0,d,1)	(1,d,1)	(2,d,0)	(0,d,2)	(2,d,1)	(1,d,2)	(2,d,2)
d	0.4899	-0.2247	0.48233	0.08569	0.001173	0.471082	0.086882	0.056475	-1.2903
AIC	8.3233	8.1165	8.2934	8.1015	8.0944	8.2288	8.1226	8.1211	8.1232
SIC	8.4237	8.2503	8.4272	8.2688	8.2617	8.3961	8.3233	8.3218	8.3574

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In the ARFIMA models under one-stage estimation, the results indicated that the best model with the lowest values in AIC was the ARFIMA (2,0.001173,0) model (AIC=8.0944). However, when comparing the SIC was found that the model with the lowest value was the ARFIMA (1,-0.22472,0) model (SIC=8.2503). Based on this, the coefficients of the selected models were estimated, and diagnostic tests were performed on the residuals. In addition, the in-sample forecasting accuracy of the selected models was evaluated, and the error functions, including RMSE and MAE, were calculated, and all these results are presented in (Table 6).

Table 6.Results of ARIMA\ ARFIMA estimation, diagnostic tests, and forecasting.

Par.	ARIMA (1,1,0)	ARFIMA (0, 0.495795,0)	ARFIMA (1,-0.22472,0)	ARFIMA (2,0.001173,0)
d	-	-	-0.2247 [0.051]	0.0011 [0.9965]
c	-	11.3773 [0.000]	62.7657 [0.001]	61.7248 [0.0071]
$\emptyset_1$	-0.3527 [0.000]	-	0.9800 [0.000]	0.6226 [0.0266]
$\emptyset_2$	-	-	-	0.32494 [0.0876]
$\hat{\sigma}^2$	161.6123[0.000]	-	168.7092 [0.000]	159.1576 [0.000]
AIC	7.9877	7.9284	8.1165	8.0944
SIC	8.0551	7.9619	8.2503	8.2617
RMSE	12.8129	12.5522	12.7437	12.5522
MAE	8.276584	8.4359	8.3018	8.0711
Diagnostic Tests				
Q(20)	-0.089 [0.932]	-0.092 [0.639]	-0.078 [0.672]	-0.080 [0.827]
$Q^2(20)$	-0.002 [0.971]	-0.019 [0.911]	-0.003 [0.740]	0.001 [0.928]
JB	259.942[0.000]	97.126 [0.000]	155.848[0.000]	187.611 [0.000]
Heteroskedasticity Test: ARCH(10)				
F-statistic	0.5474 [0.8463]	0.6224 [0.7865]	1.5693 [0.148]	0.7493 [0.675]
nR^2	6.09821 [0.8069]	6.8156 [0.7427]	14.4594 [0.153]	8.0031 [0.629]

The estimation results reported in (Table 6) indicate that most parameters in the selected models are statistically significant at the 1% level, with the exception of the parameters *d* in the ARFIMA (1,-0.22472,0) and ARFIMA(2,0.001173,0) models. Several diagnostic tests were applied to explore the suitability of the estimated models. The *p*-values of Q(20) statistics were greater than 1%, meaning that the residuals for all these models are white noise. The $Q^2(20)$ statistics for the squared residuals with *p*-values greater than 1% indicate that the squared residuals are independent and there is no ARCH effect. The JB test rejects the null hypothesis of normality for all models; however, such non-normality is commonly observed in GDP series due to economic shocks and crises. Furthermore, the heteroskedasticity tests confirm the homoskedasticity of residuals across all selected models. Based on the AIC and SIC, the ARFIMA (0,0.495795,0) model exhibits the lowest values and is therefore preferred in terms of in-sample model fit. In the final stage, in-sample forecasting performance was evaluated using RMSE and MAE. The RMSE results indicate that both ARFIMA (0,0.495795,0) and ARFIMA (2,0.001173,0) achieve the lowest error values. However, given that the fractional differencing parameter in the ARFIMA (2, 0.001173,0) model is statistically insignificant (*p*-value = 0.9965), implying the absence of long memory behaviour, the ARFIMA (0,0.495795,0) model is favored according to the RMSE criterion. In contrast, the MAE results suggest that ARFIMA (2,0.001173,0) has the lowest value, followed by the ARIMA (1,1,0) model.

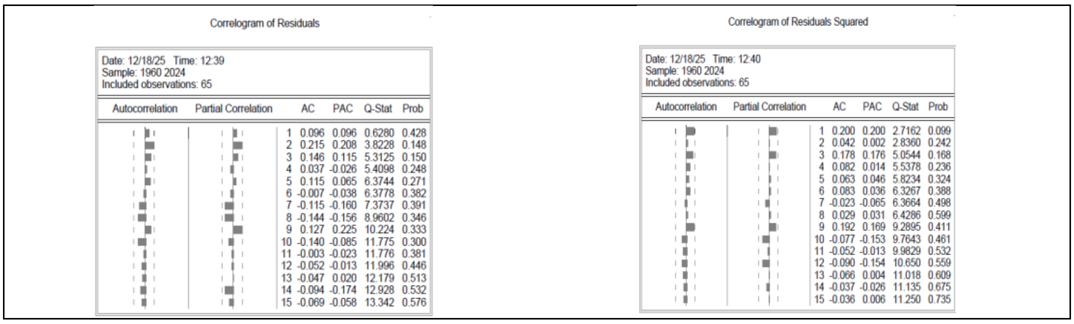

Figure 3. Correlogram of residuals and residuals squared for ARFIMA (0, 0.495795,0) model.

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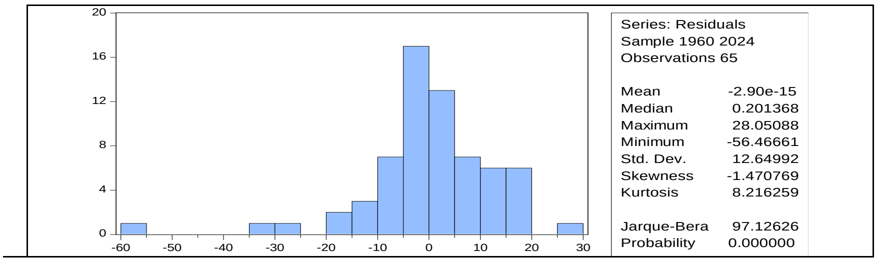

Figure 4. Histogram of residuals for ARFIMA (0, 0.495795,0) model.

Conclusion

This paper concluded that the Libyan GDP series exhibits long-memory characteristics ,with the estimated fractional difference coefficient showing a statistically significant positive value close to 0.05, reflecting the long-term persistence of economic shocks. The results confirmed that traditional ARIMA models, while capable of achieving stationarity through the first difference, do not fully capture the long-range dependency dynamics of the series. Furthermore, a comparison between ARIMA and ARFIMA models, based on information criteria and in-sample forecasting accuracy measures, demonstrated the superiority of the ARFIMA (0,0.495795,0) model in terms of fit quality and forecasting performance. Diagnostic tests showed no issues of autocorrelation and heterogeneity of variance in the residual term. The findings of this paper contribute to a practical framework that can help policymakers improve the accuracy of economic modeling and forecasting and assist governments in formulating more effective monetary policies regarding the Libyan economy. Ttherefore, the study emphasizes the importance of considering long-memory models when analyzing macroeconomic series, particularly GDP, due to their superior ability to characterize the dynamic behaviour of time series. The study suggests future expansion of the analysis through out-of-sample forecasting ability testing and investigating the impact of structural breaks and economic shocks on the long-memory characteristics of the Libyan economy.

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