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Original article

Improving the Accuracy of the Model Selection by Applying Optimal Tuning Parameters in the Elastic Net Penalized Quantile Regression Model and Empirical Mode Decomposition with Applications

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Abstract
Selecting optimal tuning parameters can enhance the accuracy of machine learning techniques, particularly when data exhibits heterogeneity and multicollinearity. Thus, this paper introduces a novel approach by combining elastic net penalized quantile regression (QRELN) with empirical mode decomposition (EMD). The EMD algorithm is used to decompose the non-stationary and nonlinear original time series predictor into a finite set of several intrinsic mode function components and one residual component. While elastic-net quantile regression (QRELN) offers more accurate estimations by addressing multicollinearity, heavy-tailed distributions, heterogeneity, and selection of the most important variables. The results of the numerical experiments and real data confirmed the superiority of the EMD. QRELN method with selecting the optimal tuning parameters. The proposed ELNET.QR αopt method also effectively identifies predictor variables that have the most significance on the response variable.
Keywords. Elastic-net Regression, Empirical Mode Decomposition, Quantile Regression, Penalized Regression, Tuning Parameters, Heterogeneity, Cross-validation.

Introduction

In several scientific fields, relationships between natural processes are frequently investigated through regression analysis using time series data. Such data are frequently non-linear and non-stationary, resulting in low-accuracy regression models and thus making the results less reliable. Non-linear and non-stationary time series data are addressed in regression through decomposing the time series data into intrinsic mode functions (IMF) through the empirical mode decomposition algorithm (EMD) [1]. Regression analysis plays a central role in statistical modelling, which is concerned with the study and relies on analyzing the relationship between a dependent variable and one or more explanatory variables. It is well known that the Ordinary Least Squares (OLS) approach is utilized to estimate the explanatory variables. However, in certain situations, it can be difficult to use due to the unavailability of one or all of its hypotheses, just as random errors are abnormal, meaning they do not follow a normal distribution [2].
Ordinary Least Squares (OLS) regression remains a foundational tool in statistical modeling and is widely used to fit the model, mainly because of tradition and ease of computation. However, its inherent assumptions, such as heterogeneity and the multicollinearity problem among the predictor variables, are often violated in real-world datasets. These violations can lead to biased estimates and unreliable predictions, thereby limiting the applicability of OLS in diverse research contexts. Consequently, researchers continuously strive to develop hybrid regression models to overcome these inherent limitations and enhance the performance of the OLS methodology [3]. In recent years, several novel regression methods have been developed that substantially enhance the ordinary least squares (OLS) method. The most popular regularization approach is penalized least squares regression. Examples include the Ridge [4], Lasso [5], SCAD [6], Elastic Net [7], Adaptive Lasso [8], and so on. However, because the least squares criteria are used, all of these approaches are not robust to outliers or heavy-tailed errors [9].
Quantile regression has been suggested as an appropriate substitute for Ordinary Least Squares. It is regarded as an extension of ordinary linear regression and a complement to the technique of least squares (OLS), which estimates the conditional distribution of the response variable at various locations. It is the strongest against outliers and unusual numbers. In addition, it minimizes the mean square error. Quantile regression is a popular technique for studying the relationship between a response variable and predictor variables at any quantile of the conditional distribution function. It provides a more comprehensive view of the phenomenon under study as it does not make any assumptions about the error term in the model [2,10]. The quantile regression (QR) is used to give a comprehensive assessment of the covariate effects on the distribution of the response variable and can model the conditional quantile of the response variable given certain covariates across various quantile levels. Therefore, quantile regression offers a more comprehensive characterization of the error distribution and yields a resilient estimate against outliers without necessitating a specific distribution for the error component [10,11].
In the penalized regression models, the tuning parameter is an important component of any penalty function in enhancing penalized least squares estimators to get consistent selection and optimal estimates. It is often employed to balance the trade-off between model fitting and model sparsity, which largely affects the numerical performance and the asymptotic behavior of the penalized regression models [12,13].

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To choose the proper tuning parameter, the existing literature offers some frequently applied approaches, such as cross-validation (CV) [14], the Akaike information criterion (AIC) [15], and the Bayes information criterion (BIC) [16]. The cross-validation method is the simplest and most popular method for estimating the prediction error and selecting the tuning parameter with the minimum sum of squared residuals [17]. The CV technique involves presenting a grid of λ values and calculating the CV error for each λ after selecting the optimal λ with the lowest CV error [18,19].
The contribution of this paper is threefold. First, the current study aims to solve the problem of multi-scale time series data in regression by decomposing the series time data into intrinsic mode functions (IMF) through the empirical mode decomposition algorithm (EMD) [1]. The resulting IMFs represent the basic oscillation modes of time series data and can be utilized as variables in regression analyses. Secondly, to enhance the prediction accuracy of the model selection by choosing the predictor variables that have the most effect on the response variable. Subsequently, it is evaluated and compared with recently developed methods by both simulations and practical applications. Finally, with regard to the novelty of the econometrics aspect, we model and predict the daily closing of the stock market prices to address heterogeneous time series data. The rest of the article is organized as follows. In Section 2, after the introduction of some notation and the model.

Methods

This section briefly describes the methods used: elastic-net penalized quantile regression, empirical mode decomposition via a sifting process for signal decomposition, and optimal tuning parameter selection. This section also discusses the proposed EMD.QRELN method.

Penalized quantile regression

Quantile regression describes the relationship between the response variable and the predictor variables at any different quantile of the conditional distribution of the response variable$Q_\tau(y/X)$.Where τ represents the quantiles or percentiles and ranges from 0 to 1.
The multiple linear regression model is given by,

$$y = X\beta + \varepsilon \tag{1}$$

Where $y = (y_1, y_2, \dots \dots . y_n)^T$ is a vector of the response variable, $X = (x_1, \dots \dots . . x_n)^T$ is a matrix of the predictor variables, $\beta = \left(\beta_1, \dots \dots \dots \dots \beta_p\right)^T$ is a vector of unknown regression coefficients, and is $\varepsilon = \left(\varepsilon_1, \dots \dots \dots \dots \varepsilon_p\right)^T$ is a vector of the random observation errors that are supposed to be normally distributed with and $E(\varepsilon) = 0$ and $, E(\boldsymbol{\varepsilon\varepsilon^T}) = \sigma^2 \mathbf{I_n}$.Where $y = (y_1, y_2, \dots \dots . y_n)^T$ is a vector of the response variable, $X = (x_1, \dots \dots . . x_n)^T$ is a matrix of the predictor variables, $\beta = \left(\beta_1, \dots \dots \dots \beta_p\right)^T$ is a vector of unknown regression coefficients, and $\varepsilon = \left(\varepsilon_1, \dots \dots \dots \dots \varepsilon_p\right)^T$ is a vector of the random observation errors that are supposed to be normally distributed with $E(\varepsilon) = 0$ and $, E(\boldsymbol{\varepsilon\varepsilon^T}) = \sigma^2 \mathbf{I_n}$.

The linear quantile regression model assumes

$$Q_\tau(y/X) = X\beta \tag{2}$$

Where $Q_\tau(y/X)$ is the conditional quantile function for the τ-th conditional quantiles or percentiles with. $(0 < \tau < 1)$. The τ-th quantile regression estimator β minimizes the objective function. Using the following formula [20].

$$\hat{\beta}_\tau = \min_\beta \sum_{i=1}^{n} \rho_\tau \left(y_i - x_i^T \beta\right) \tag{3}$$

Where $\rho_\tau(u)$ the loss function is defined as follows: Where $\rho_\tau(u)$ the loss function defined as follows:
Under the regularization and to improve quantile regression,$\rho_\tau(u) =$

$$\begin{cases} \tau u & if\ \ u > 0 \\ (\tau - 1)u & if\ \ u \le 0 \end{cases} \tag{4}$$

Koenker proposed a penalized version of the following: Under the regularization and to improve quantile regression, Koenker proposed penalized version of the following:

$$\hat{\beta}_\tau = \min_\beta \sum_{i=1}^{n} \rho_\tau \left(y_i - x_i^T \beta\right) + P_\lambda(\beta) \tag{5}$$

Where is the tuning parameter, and $P_\lambda(\beta)$ represents the penalty function.

Elastic net penalized quantile regression

The elastic-net regression is proposed by [7] to deal with the limitations of LASSO and enhance the interpretability of the model and accuracy prediction through combining the advantages of LASSO penalty and ridge penalty, which can effectively solve the problems of continuous contraction and automatic variable selection [10]. An elastic-net penalized quantile regression with applications [19]. The elastic net estimator is as follows:

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$$\hat{\beta}_{Enet} = \min_{\beta}(y_i - x_i^T \beta) + \lambda_1\|\beta\|_1 + \lambda_2\|\beta\|_2^2 \quad (6)$$

Where $\|\beta\|_2^2 = \sum_{j=1}^{p}\beta_j^2$ is the L_2-norm square of $\|\beta\|_2^2 = \sum_{j=1}^{p}\beta_j^2$e vector β, and $\|\beta\|_1 = \sum_{j=1}^{p}|\beta_j|$ is the L_1-norm of the vector β. Where λ_1 and λ_2 are tuning parameters, ($\lambda_1,\ \lambda_2 \geq 0$), where these parameters control the amount of shrinkage for regression parameters, and they are automatically selected via cross-validation (CV) [7,$L_2$0,19]. By denoting $\lambda_1 = 2n\lambda\alpha$ and $\lambda_2 = n\lambda(1-\alpha)$ then equation (2.31) is equivalent to the optimization problem as follows:-norm square of the vector β and $\|\beta\|_1 = \sum_{j=1}^{p}|\beta_j|$ is the L_1-norm of the vector β. Where λ_1 and λ_2 are tuning parameters ($\lambda_1,\ \lambda_2 \geq 0$), where these parameters control the amount of shrinkage for regression parameters, and they are which are automatically selected via cross-validation (CV) [7,10,19]. By denoting $\lambda_1 = 2n\lambda\alpha$ and $\lambda_2 = n\lambda(1-\alpha)$ then equation (2.31) is equivalent to the optimization problem as the following:

$$\hat{\beta}_{Enet} = \min_{\beta}\sum_{i=1}^{n}(y_i - x_i^T \beta) + \lambda\sum_{j=1}^{p}\left(\alpha\|\beta\|_1 + \frac{(1-\alpha)}{2}\|\beta\|_2^2\right) \quad (7)$$

Where $\alpha \in [0,1]$ is a regularization parameter, when $\alpha = 0\alpha \in [0,1]$a ridge penalty $\sum_{j=1}^{p}\beta_j^2$ and when $\alpha = 1$, we get a LASSO penalty $\sum_{j=1}^{p}|\beta_j|$. is a regularization parameter, when $\alpha = 0$, we get a ridge penalty $\sum_{j=1}^{p}\beta_j^2$ and when $\alpha = 1$, we get LASSO penalty $\sum_{j=1}^{p}|\beta_j|$.

The elastic-net penalized quantile regression can be obtained by employing the quantile loss along with the Elastic-net penalty. The estimator for the elastic-net penalized quantile regression is as follows:

$$\hat{\beta}_{QREnet} = \min_{\beta}\sum_{i=1}^{n}\rho_\tau(y_i - x_i^T\beta) + \lambda\left[\alpha\|\beta\|_1 + \frac{(1-\alpha)}{2}\|\beta\|_2^2\right] \quad (8)$$

Empirical mode decomposition

[1] suggested a novel empirical mode decomposition (EMD) technique for decomposing non-stationary and nonlinear signals into a finite number of intrinsic mode functions (IMFs) and residual components via the sifting process [21]. In this method, the time domain of the signals is unchanged. Each IMF must fulfil the following two conditions [22]:

1. The number of extrema values and zero crossings are equal or differs by at most one.
2. The mean value of the envelope defined by the mean of the upper and lower envelopes must be zero. This ensures that each IMF represents a distinct oscillatory mode within the original signal.

Satisfying these conditions secures that narrowband (single scale) IMFs permit the representation in (1) while also lending themselves to transmitting physically meaningful information [23]. Algorithm 1 summarizes the steps for the extraction of IMFs from the original signal $x(t)$.

Algorithm 1 Empirical Mode Decomposition

Input: $x(t)$

1) Initialization$x(t) = r_0(t), q = 1\ and\ k = 1$.
2) Identify all local maxima and minima.
3) By using cubic spline interpolation, connect all local extrema to generate the upper envelope $U_q(t)$ and lower envelope $L_q(t)$, respectively.
4) Find the mean envelope as: $m_q(t) = \frac{U_q(t)+L_q(t)}{2}$
5) Extract $h_q(t)$ by using the relation:$h_q(t) = x_1(t) - m_q(t)$
6) Check if the $h_q(t)$ satisfies the IMF conditions:
   - **Yes,** then $h_q(t) = C_k(t)$, save the output $C_k(t)$, and go to (7).
   - **No:** Let $q = q + 1$**,** and repeat the procedures from step 2.
7) Calculate: $r_k(t) = r_{k-1}(t) - C_k(t)$
8) Check if $r_k(t)$ is a monotonic or constant function or satisfies the stopping criterion of the standard deviation: $SD_q = \sum_{t=0}^{T}\frac{h_{q-1}(t)-h_q(t)^2}{h_{q-1}^2(t)}$

(A typical value for can be set between 0.2 and 0.3)SD_q can be set between 0.2 and 0.3)

- **Yes:** Save $r(t)$, the sifting process stops.
- **No:** continue the decomposition from step 2, setting $k = k + 1$.

Output:

$C_{1k}(t)\ and\ r_1(t), \quad k = 1,2 \ldots \ldots K$

The original signal $x(t)$ is expressed as the linear combination of the finite set of orthogonal IMF components and one residual component by the EMD algorithm, as indicated by the following Equation 1:

$$x(t) = \sum_{k=1}^{K} C_k(t) + r(t) \quad (9)$$

Where t represents the sample index (time domain), intrinsic mode functions (IMFs) are denoted by $\{C_k(t), k = 1,2, \ldots . . K\}$ and $r(t)$ is the residual component.

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Choice of Optimal Tuning Parameter

The tuning parameter λ plays an important role in the optimization issue for the mentioned penalized least squares estimators to obtain consistent selection and optimum estimate [13]. The Q-fold cross-validation algorithm is described in detail as follows: Firstly, the dataset is randomly split into Q folds of equal length. One of the Q-1 folds is assigned the role of the training set for the estimation of the model, while the remaining Q folds together constitute the test set to assess the predictive performance of the model. In practice, the number of Q folds is commonly set at 5 or 10. The process repeats until every Q-fold is used as the test set. Then, the MSE across all folds is determined [19,24].

$$MSE_{i,\lambda_s} = \frac{1}{Q}\sum_{q=1}^{Q} RSS_{i,\lambda_s} \tag{10}$$

where $RSS_{i,\lambda_s} = \sum_{i=1}^{n}\left(y_i - \hat{f}_{d,\lambda_v}(x_i)\right)^2$

The optimal λ that gives the minimum MSE_{i,λ_s} is expressed as follows:

$$\mathrm{CV}_{\lambda_s} = \frac{1}{Q}\sum_{q=1}^{Q} MSE_{\mathrm{i},\lambda_s}$$

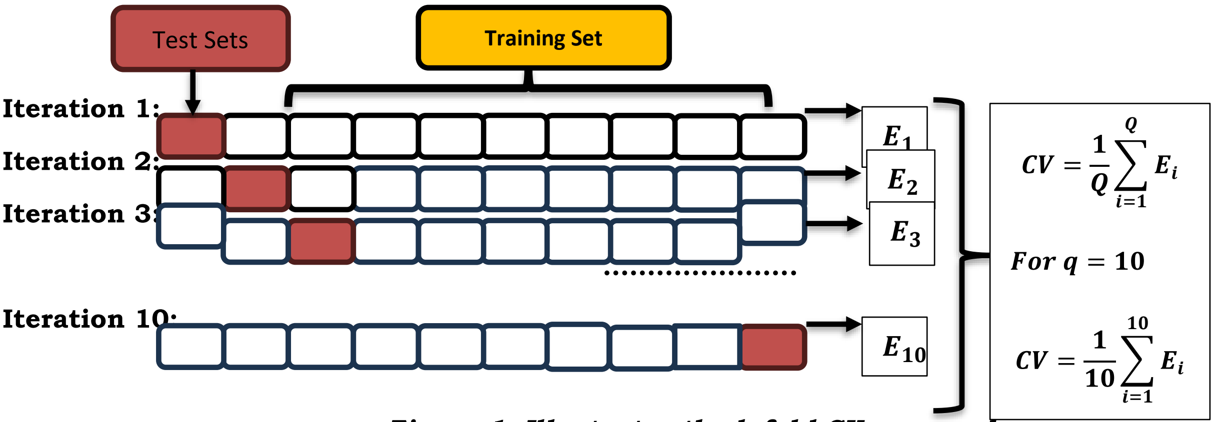

Figure 1. Illustrates the k-fold CV approach.

(Figure 1) displays the procedure of Q-fold cross-validation. A set of n observations is divided randomly into five non-overlapping groups. Each of these fifths acts as a validation set (shown in beige), and the remainder as a training set (shown in blue). The test error is estimated by taking the average of the five resulting MSE estimates.

When the Q value increases leads to a reduction in the bias of the fit model, whereas the variance will rise, as will the fitted model's correlation, due to the overlap between the training sets [18]. In practice, the Q value is typically chosen by using Q =5 or Q =10, and these values yield estimates with an intermediate level of bias that is neither highly biased nor significantly variable. On. As a result, Q =5 or 10 includes the bias-variance trade-off [18,25].

Proposed Elastic-net Penalized Quantile Regression Method

To explain the significance of the predictor variables on the response variable and enhance the prediction error of the final model based on the optimal tuning parameters, we propose the following three-step Elastic-net penalized quantile regression method based on EMD using 10-CV as follows:

1. The original signals x_j (t) are decomposed by EMD into several components, named $C_{jk}(t)$ and the residual component $r_j(t)$. These decomposed components are as follows.

$$x_j(t) = \sum_{k=1}^{K} C_{jk}(t) + r_j(t)\,;\, j = 1,2, \ldots \ldots . p \tag{11}$$

2. All the decomposition components obtained in Step 1 are used as predictor variables to predict the behavior of the response variable as follows:

$$y(t) = \sum_{j=1}^{p}\left[\sum_{k=1}^{K} C_{jk}\beta_{jk} + r_{jk}(t)\beta_{jk}\right] + \varepsilon(t) \tag{12}$$

3. Select the optimal parameters via the Q-VD method at Q =10, using the training set only, as follows:
   - The value of the regularization parameter α_{Optaml} in the sequence, $0 < \alpha < 1$, where αopt denotes the relative contribution of the L_1 penalty versus theL_2 penalty.

$$\alpha_{optmal} = \underset{\alpha_s \in (0,1)}{argmin}\left(MSE_{\alpha_s}\right) \tag{13}$$

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$$MSE_{\alpha_s} = \frac{1}{10}\sum_{q=1}^{10} RSS_{q,\alpha_s}$$

- Where k is the number of α values between $0 < \alpha <$ that will be selected. In this study, we choose K = 50.0 $< \alpha <$ that will be selected. In this study, we choose K = 50.

$$\lambda_{optamal} = \underset{s=1,2,\ldots,S}{argmin}\left(MSE_{\lambda_s}\right) \quad (14)$$

$$MSE_{\lambda_s,\alpha_{opt}} = \frac{1}{10}\sum_{q=1}^{10} RSS_{q,\lambda_s,\alpha_{opt}}$$

Based on the training dataset, the Equations (7) and (10) at α_{opt} and λ_{opt}, the ELN penalized regression is used follows formula: and λ_{opt}, the ELN penalized regression is used as the following formula:

$$\min_{\beta}\left[\frac{\rho_\tau}{n}\left(y(t) - \sum_{j=1}^{p}(\sum_{k=1}^{K} C_{jk}(t)\,\beta_{jk} - r_j(t)\beta_{jk+1})\right)^2\right] + \lambda_{opt}P(\beta) \quad (15)$$

$$;\lambda_{opt}P(\beta) = \lambda_{opt}\left(\alpha_{opt}\sum_{k=1}^{p}\left[\sum_{k=1}^{K}\beta_{jk}^2\right] + \frac{(1-\alpha_{opt})}{2}\sum_{k=1}^{p}\left[\sum_{k=1}^{K+1}\beta_{jk}^2\right]\right)$$

Finally, an evaluation was performed between the proposed methods and traditional methods. Several well-known criteria were employed to evaluate the efficacy of the proposed estimated method. The test criteria include root mean square error (RMSE), mean absolute error (MAE), mean absolute scaled error (MASE), and mean absolute percentage error (MAPE).

$$RSS = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \quad (16)$$

$$MAPE = \frac{100\%}{n}\sum_{i=1}^{n}\frac{y_i - \hat{y}_i}{y_i} \quad (17)$$

$$MASE = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{|y_i - \hat{y}_i|}{\frac{1}{n-1}\sum_{i=2}^{n}|y_i - y_{i-1}|}\right) \quad (18)$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i| \quad (19)$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} \quad (20)$$

$$PE = \frac{1}{n}\sum_{i=1}^{n}\rho_\tau(y_i - \hat{y}_i) \quad (21)$$

NUMERICAL STUDIES

In this section, we implement a simulation experiment and a real dataset application to evaluate the performance of the proposed method. The function of obtaining the optimal tuning parameter value for EMD.QRELN is calculated using the hqreg package and our developed code, and the analyses are performed using open-source R software.

Simulation Study

This section presents the results of the numerical simulation for the proposal methods and traditional methods, namely EMD. QRRidge, EMD.QRLASSO, EMD.QRELN at the optimal α value; EMD.QRELN at α = 0.25, EMD.QRELN at α = 0.5, EMD.QRELN at α = 0.75 based on the minimum MSE and minimum MSE with one standard error (λ_{1se}) of weighted RR to evaluate and illustrate the performance of these methods of variable selection and prediction. The simulation scenarios were conducted with a sample size of 150, an iteration of 1000, under three different quantiles (τ = 0.25, 0.5, and 0.75). The optimal tuning parameter values were chosen based on 10-fold cross-validation. The simulated datasets were split into two sections: 70% for training the model and 30% for testing and assessing performance criteria.MSE (λ_{min}) and minimum MSE with one standard error (λ_{1se}) of weighted RR to evaluate and illustrate the performance of these methods of variable selection and prediction. The simulation scenarios were conducted with a sample size of 150, an iteration of 1000, under three different quantiles (τ = 0.25, 0.5, and 0.75). The optimal tuning parameter values were chosen based on 10-fold cross-validation. The simulated datasets were split into two sections: 70% for training the model and 30% for testing and assessing performance criteria.

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Real data application

In this section, we applied the daily close exchange rates from 27/03/2015 to 25/10/2019 of three countries against the US dollar (USD). The dataset selected three countries: Japan (JAP), China (CHN), and Taiwan (TAW). The datasets were collected from the Wall Street Journal database (https://www.wsj.com/). The datasets were divided into two parts: 70% for the training model and 30% for testing and assessing their performance criteria.

Where y represents the daily close exchange rates of TWA, x_1 is the daily close exchange rates of JAP, and x_2 is the daily close exchange rates of CHA?

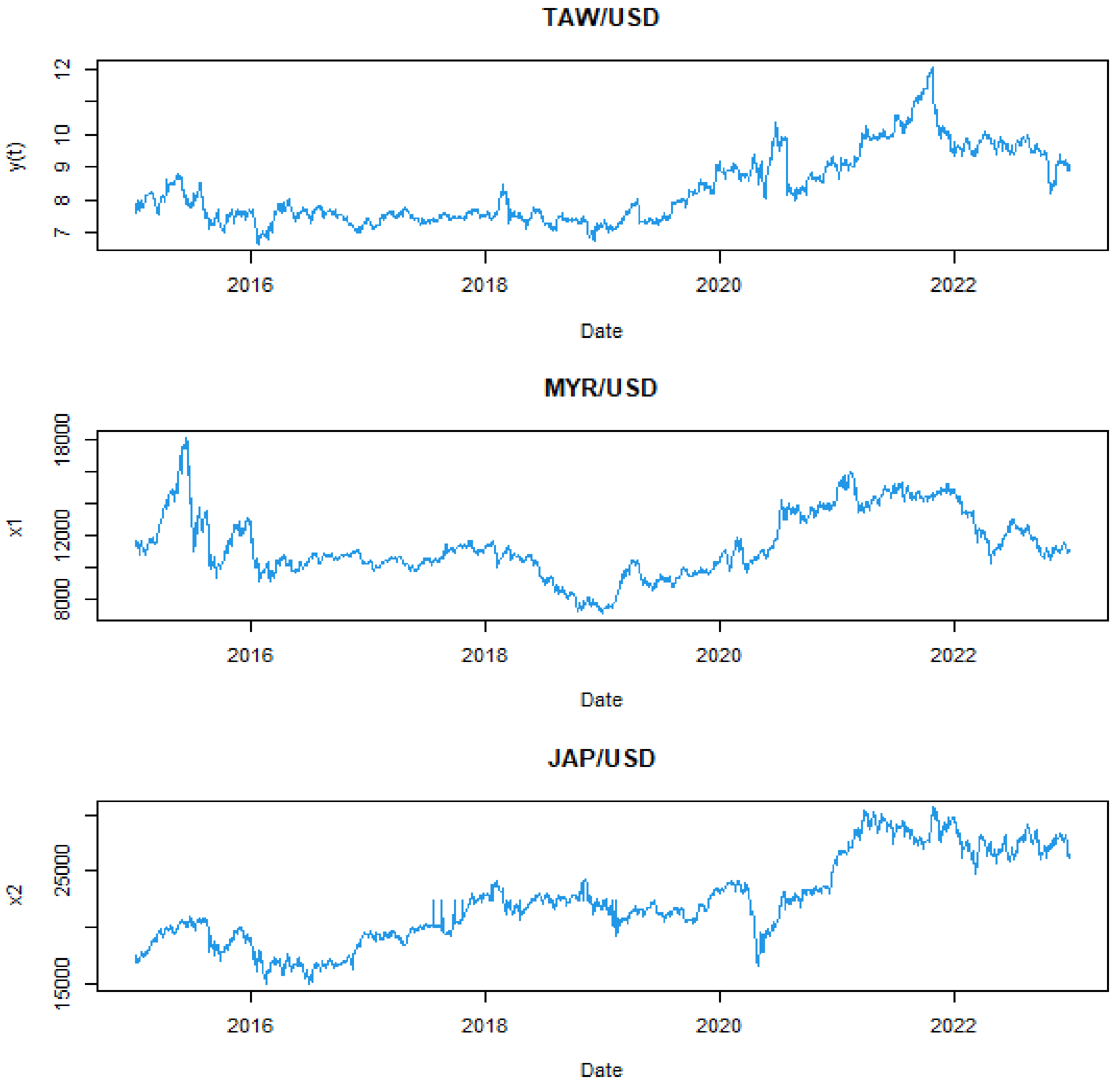

Figure 2. The daily stock market Index is plotted over time

Results and discussion

This section presents the results of a simulation experiment and the application based on the real dataset.

Simulation Results

(Table 1) presents the mean of the performance criteria in terms of the MAE, RMSE, MASE, and MAPE used in this study for all regression methods at three different quantiles (τ = 0.25, 0.5, and 0.75). The results show that the proposed method, EMD.QRELN with α_{obt} (by identifying the optimal α value) and λ_{min}, outperformed all the existing methods because they have the smallest values in these criteria tests. Therefore, the prediction accuracy is enhanced by ELNET.QR αopt, λmin, which provides the minimum error values in terms of RSS, RMSE, MAE, MASE, and MAPE., outperformed all the existing methods because they have the smallest values in these criteria tests. Therefore, the prediction accuracy is enhanced by ELNET.QR αopt, λmin, which provides the minimum error values in terms of RSS, RMSE, MAE, MASE, and MAPE.

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Table 1. Mean performance criteria of the simulation scenarios.

Methods	λ	EP	RMSE	MASE	MAE	MAPE
	$\tau = 0.25$					
EMD. QRRidge	λ_{min}	0.1354747	1.016512	0.7860301	0.8268522	6.608568
	λ_{1se}	0.1478563	1.078948	0.8336665	0.8769640	6.557723
EMD. QRLasso	λ_{min}	0.1394191	1.019281	0.7879936	0.8289927	6.699076
	λ_{1se}	0.1500528	1.074465	0.8297050	0.8728321	6.568887
EMD.QRELN $\alpha = 0.25$	λ_{min}	0.1371068	1.017489	0.7867084	0.8276115	7.219920
	λ_{1se}	0.1460451	1.054101	0.8137576	0.8560514	6.698597
EMD.QRELN $\alpha = 0.50$	λ_{min}	0.1380138	1.020055	0.7881625	0.8291776	6.877224
	λ_{1se}	0.1462457	1.052600	0.8125428	0.8547959	6.961208
EMD.QRELN $\alpha = 0.75$	λ_{min}	0.1397449	1.026421	0.7928249	0.8340972	6.700799
	λ_{1se}	0.1471747	1.057723	0.8164515	0.8588681	7.170749
EMD.QRELN $\alpha_{opt} = 0.96$	λ_{min}	0.1362572	1.009522	0.7798265	0.8204369	6.640818
	λ_{1se}	0.1467088	1.054956	0.8143385	0.8566673	6.887441
$\tau = 0.50$						
EMD. QRRidge	λ_{min}	3.032463e-03	0.859718	0.6510376	0.6851058	2.918828
	λ_{1se}	2.146358e-03	0.913019	0.6943930	0.7305373	1.761690
EMD. QRLasso	λ_{min}	3.932728e-03	0.848583	0.6468290	0.6807689	3.324123
	λ_{1se}	2.879077e-03	0.886123	0.6756263	0.7109228	2.328907
EMD.QRELN $\alpha = 0.25$	λ_{min}	4.241935e-03	0.851485	0.6493875	0.6833685	3.477430
	λ_{1se}	2.101237e-03	0.885263	0.6729727	0.7080891	2.156452
EMD.QRELN $\alpha = 0.50$	λ_{min}	3.897117e-03	0.853686	0.6507148	0.6847823	3.284174
	λ_{1se}	2.326496e-03	0.882947	0.6720231	0.7071102	2.255543
EMD.QRELN $\alpha = 0.75$	λ_{min}	3.801264e-03	0.858101	0.6540390	0.6883073	3.134581
	λ_{1se}	2.489620e-03	0.954355	0.7277213	0.7676498	1.418677
EMD.QRELN $\alpha_{opt} = 0.86$	λ_{min}	4.105784e-03	0.846924	0.6447679	0.6785803	3.265480
	λ_{1se}	2.255413e-03	0.883379	0.6723447	0.7075054	2.232432
$\tau = 0.75$						
EMD. QRRidge	λ_{min}	-0.3970951	1.017384	0.7918248	0.8343655	5.136426
	λ_{1se}	-0.4301894	1.075527	0.8345374	0.8793584	4.867550
EMD. QRLasso	λ_{min}	-0.4116700	1.018935	0.7933204	0.8360795	5.323473
	λ_{1se}	-0.4398064	1.069951	0.8309145	0.8755198	4.936393
EMD.QRELN $\alpha = 0.25$	λ_{min}	-0.4068918	1.019832	0.7937584	0.8365548	5.450387
	λ_{1se}	-0.4261248	1.051185	0.8157785	0.8595705	4.835119
EMD.QRELN $\alpha = 0.50$	λ_{min}	-0.4095664	1.021791	0.7946008	0.8375085	5.333350
	λ_{1se}	-0.4269654	1.050048	0.8150850	0.8588723	4.892463
EMD.QRELN $\alpha = 0.75$	λ_{min}	-0.4143497	1.028696	0.7995473	0.8427360	5.217303
	λ_{1se}	-0.4303024	1.054455	0.8187435	0.8627467	4.954138
EMD.QRELN $\alpha_{opt} = 0.88$	λ_{min}	-0.4019126	1.008797	0.7847176	0.8269984	5.187865
	λ_{1se}	-0.4285027	1.051860	0.8164825	0.8603981	4.901385

Table 2 presents the simulation results, including bias, RSS, the optimal λ selected via 10-fold cross-validation, and the number of variables selected for both the proposed and existing methods. The RSS values indicate that EMD-QRELN consistently achieves the lowest bias and RSS across all quantile levels. For instance, at τ = 0.25, EMD-QRELN achieves (λmin = 0.02022903; RSS = 76.61081; Bias = 0.2989208; V.S= 6); at τ = 0.5 (λ_{min} = 0.03337871; RSS = 54.19229; Bias=0.00127988; V.S = 10); and at τ = 0.75 (λ_{min} = 0.01296830; RSS = 76.82955; Bias=0.2892633; V.S = 9).

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Table 2. Coefficients estimation for the predictor variables and RSS error values.

Methods	λ	Bias	RSS	V.S
		$\tau = 0.25$		
EMD. QRRidge	λ_{min}=0.08322968	0.2953428	77.65669	$x_1, x_2, \dots x_{15}$
	λ_{1se}=0.26282306	0.3509044	87.41933	$x_1, x_2, \dots x_{15}$
EMD. QRLasso	λ_{min}=0.01161013	0.3130467	78.11064	x_2, x_3, x_4, x_8, x_9
	λ_{1se}=0.05600237	0.3619752	86.80109	x_2, x_3, x_4, x_9
EMD.QRELN $\alpha = 0.25$	λ_{min}=0.04933789	0.3028717	76.42391	$x_1, x_2, .. x_{10}, x_{12} \dots x_{15}$
	λ_{1se}=0.09895552	0.3428099	83.48341	x_2, x_3, x_4, x_9
EMD.QRELN $\alpha = 0.50$	λ_{min}=0.02784309	0.3070468	76.47008	$x_1, .. x_5, x_7 \dots x_{10}$
	λ_{1se}=0.05755971	0.3437469	83.25182	x_2, x_3, x_4, x_9
EMD.QRELN $\alpha = 0.75$	λ_{min}=0.02032597	0.3151610	76.77440	$x_1, .. x_5, x_7 \dots x_9$
	λ_{1se}=0.03955206	0.3484254	84.09256	x_2, x_3, x_4, x_9
EMD.QRELN $\alpha_{opt} = 0.96$	λ_{min}=0.02022903	0.2989208	76.61081	$x_2, x_3, x_4, x_5, x_8, x_9$
	λ_{1se}=0.03487594	0.3459302	83.62594	x_2, x_3, x_4, x_9
		$\tau = 0.50$		
EMD. QRRidge	λ_{min}=0.02211726	0.00114157	55.81352	$x_1, x_2, \dots x_{15}$
	λ_{1se}=1.81807884	0.00078994	62.84169	$x_1, x_2, \dots x_{15}$
EMD. QRLasso	λ_{min}=0.02701995	0.00135473	54.41795	$x_1, \dots x_8, x_{10} \dots x_{12}, x_{14}$
	λ_{1se}= 0.08031309	0.00098826	59.26353	x_3, x_4, x_{10}, x_{11}
EMD.QRELN $\alpha = 0.25$	λ_{min}=0.07075555	0.00138549	54.35201	$x_1, \dots x_8, x_{10} \dots, x_{14}$
	λ_{1se}= 0.29337371	0.00092650	59.14409	x_3, x_4, x_{10}, x_{11}
EMD.QRELN $\alpha = 0.50$	λ_{min}=0.04787929	0.00133206	54.19250	$x_1, \dots x_8, x_{10} \dots x_{12}, x_{14}$
	λ_{1se}=0.17064744	0.00096175	58.83910	x_3, x_4, x_{10}, x_{11}
EMD.QRELN $\alpha = 0.75$	λ_{min}=0.03827425	0.00128200	54.13750	$x_1, \dots x_8, x_{10} \dots x_{12}, x_{14}$
	λ_{1se}=0.10389230	0.00096091	59.31439	x_3, x_4, x_{10}, x_{11}
EMD.QRELN $\alpha_{opt} = 0.86$	λ_{min}=0.03337871	0.00127988	54.19229	$x_1, \dots x_6, x_8, x_{10}, .. x_{12}$
	λ_{1se}= 0.09625640	0.00099103	58.90156	x_3, x_4, x_{10}, x_{11}
		$\tau = 0.75$		
EMD. QRRidge	λ_{min}=0.10421747	0.2822218	78.15396	$x_1, x_2, \dots x_{15}$
	λ_{1se}=0.58481486	0.3302069	87.27370	$x_1, x_2, \dots x_{15}$
EMD. QRLasso	λ_{min}=0.03946173	0.3034389	78.41482	x_2, x_3, x_4
	λ_{1se}=0.08157880	0.3458979	86.53654	$x_2, \dots, x_4$
EMD.QRELN $\alpha = 0.25$	λ_{min}=0.02417963	0.2966097	76.69255	$x_2, .. x_5, x_7, x_{9,\dots} x_{13}, x_{15}$
	λ_{1se}=0.14414876	0.3243388	83.40657	x_2, x_3, x_4
EMD.QRELN $\alpha = 0.50$	λ_{min}=0.02499314	0.3009045	76.73107	$x_2, .. x_5, x_7, .., x_9, x_{12} .. x_{15}$
	λ_{1se}=0.11009433	0.3256795	83.23282	x_2, x_3, x_4
EMD.QRELN $\alpha = 0.75$	λ_{min}=0.01476259	0.3084323	77.07654	$x_2, .. x_5, x_{7,} .., x_9, x_{13} .. x_{15}$
	λ_{1se}=0.05938567	0.3312063	83.98675	x_2, x_3, x_4
EMD.QRELN $\alpha_{opt} = 0.88$	λ_{min}=0.01296830	0.2892633	76.82955	$x_2, .. x_4, x_7, \dots x_9, x_{13} .. x_{15}$
	λ_{1se}=0.06255360	0.3281261	83.52446	x_2, x_3, x_4

Application results

(Figure 3) illustrates the RSS curves for determining the optimal alpha (αopt) at τ values of 0.25, 0.50, and 0.75. The y-axis represents the estimated RSS, and the x-axis represents alpha values. The minimum RSS occurs at αopt = 0.96 when τ = 0.25 and at αopt = 0.92 when τ = 0.50 and 0.75. These results suggest that using the optimal alpha minimizes RSS more effectively than traditional methods employing fixed alpha values (e.g., 0.25, 0.5, 0.75) or methods like lasso and ridge regression.

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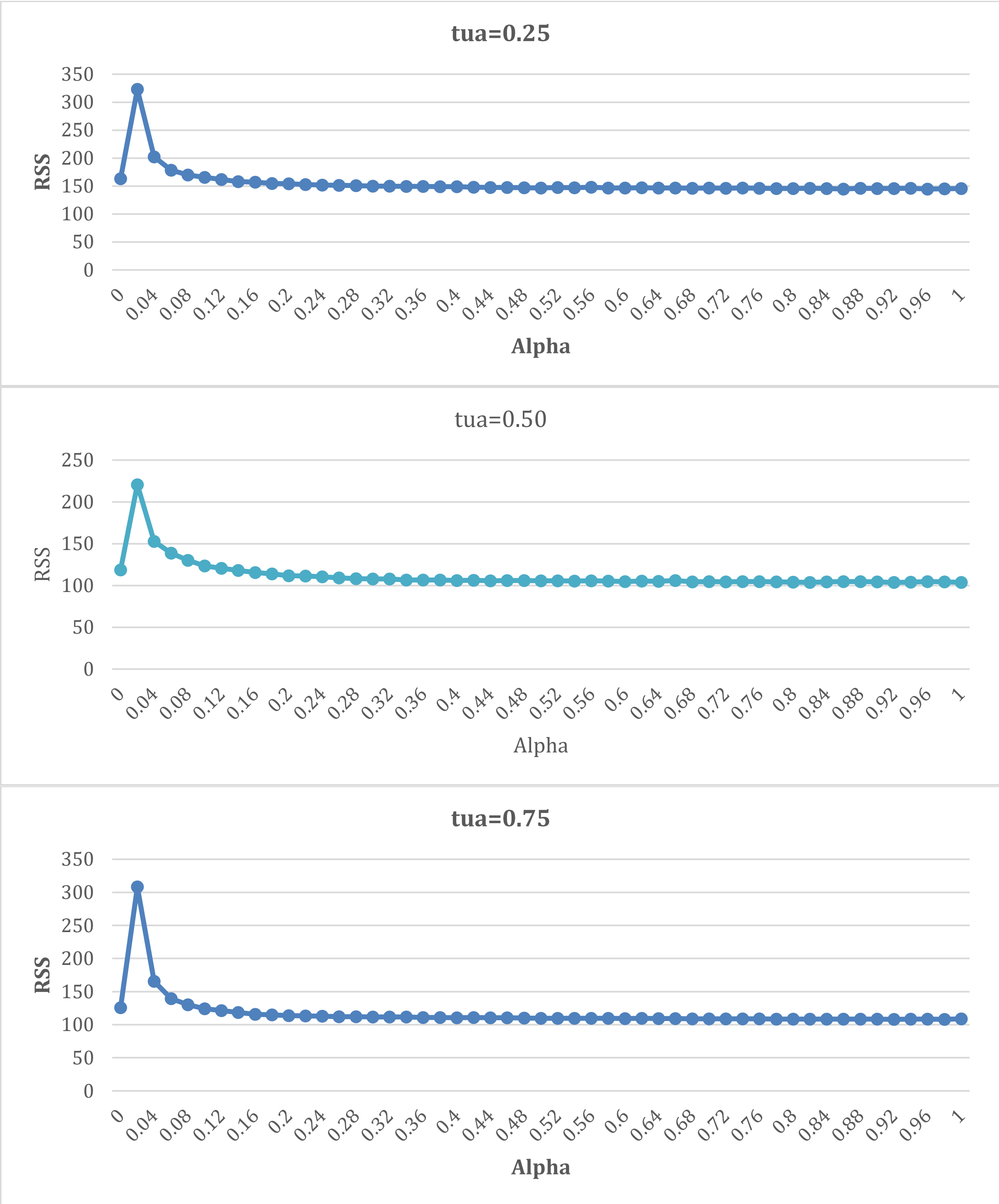

Figure 3. 10-CV estimate plot for choosing α_{Opt} of the EMD.QRELN (τ = 0.25, 0.50, 0.75).

Figure 4 shows the 10-CV estimate plot of the EMD.QRELN with α_{Opt} (α_{Opt} = 0.96, 0.92, and 0.92) and (τ = 0.25, 0.50, and 0.75), respectively. The grey bars at each point represent $MSE\lambda$ plus and minus one standard error. The mean square error (MSE) curve is shown by the red dotted line, which has one standard deviation band along the error bars. The y-axis represents the mean square error (MSE), whereas the x-axis represents the log (λ). The upper horizontal line represents the nonzero coefficients selected at the log (λ) value. The first vertical dotted line from the left indicates the point picked at minimal MSE, while the second vertical line indicates the point selected at minimum MSE using the one-standard-error (1se) criterion. The CV plot shows that increasing log(λ) leads to a reduction in the number of non-zero coefficients entering the final model.

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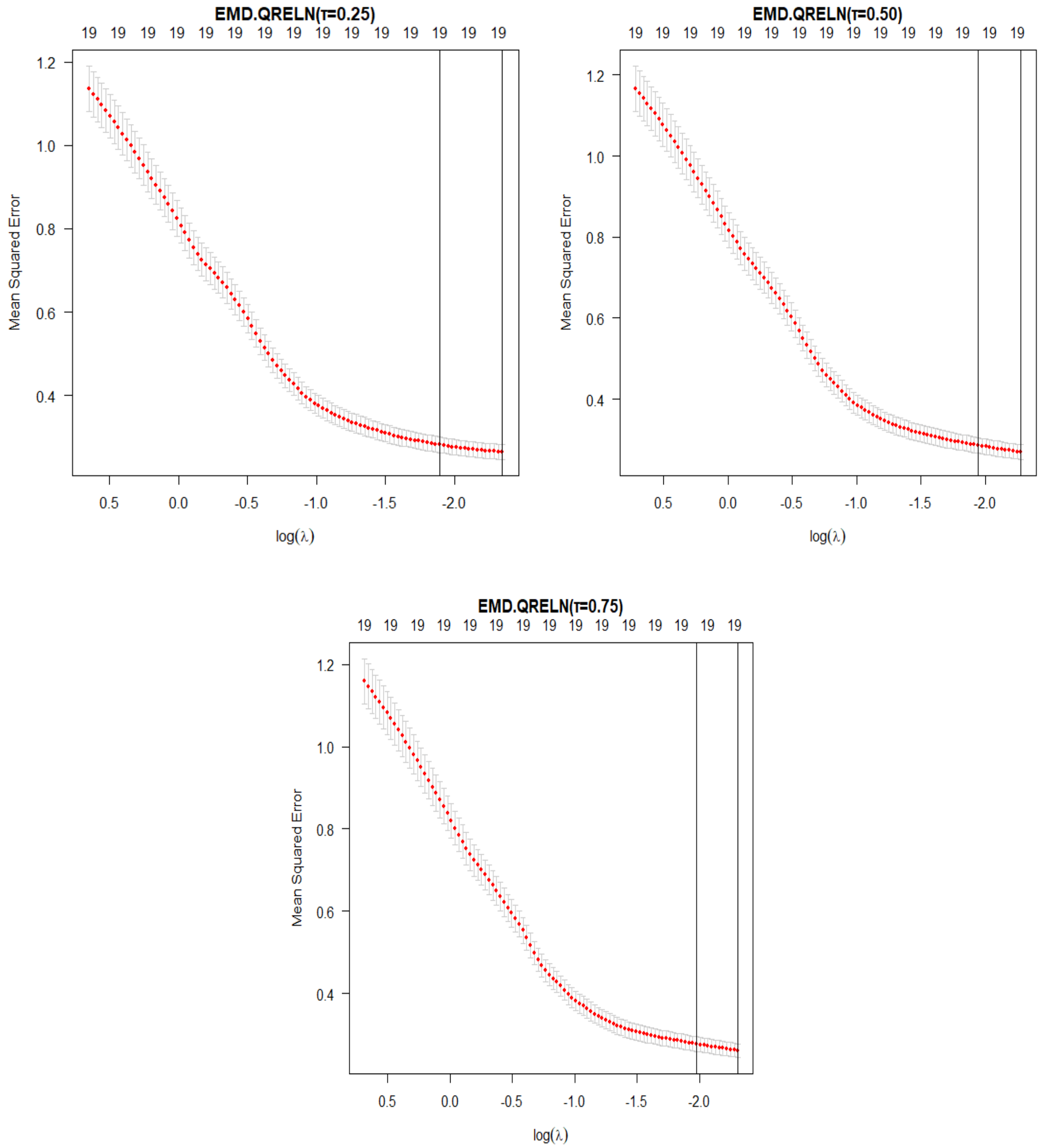

Figure 4. 10-CV estimate of the MSE for the EMD.QRELN α_{opt} at τ = 0.25, 0.50, 0.75.

Figure 5 shows the relationship between log(λ) and the selected nonzero coefficient estimation for the EMD.QRELN method at (0.96, 0.92, 0.92). The top part of the figure shows the number of non-zero coefficients in the regression model as a function of $log\ (\lambda_{min})$. In each figure (from right to left), coefficient estimates shrink towards zero as λ increases. Effectively forcing unnecessary coefficients to zero (i.e., as $\lambda \rightarrow \infty$, estimated coefficients $\rightarrow$ 0). For example, at τ = 0.25, the EMD.QRELN model with α = 0.96 selected six nonzero coefficients at one λ_{min} value and four at λ1se, indicating varying degrees of significance.

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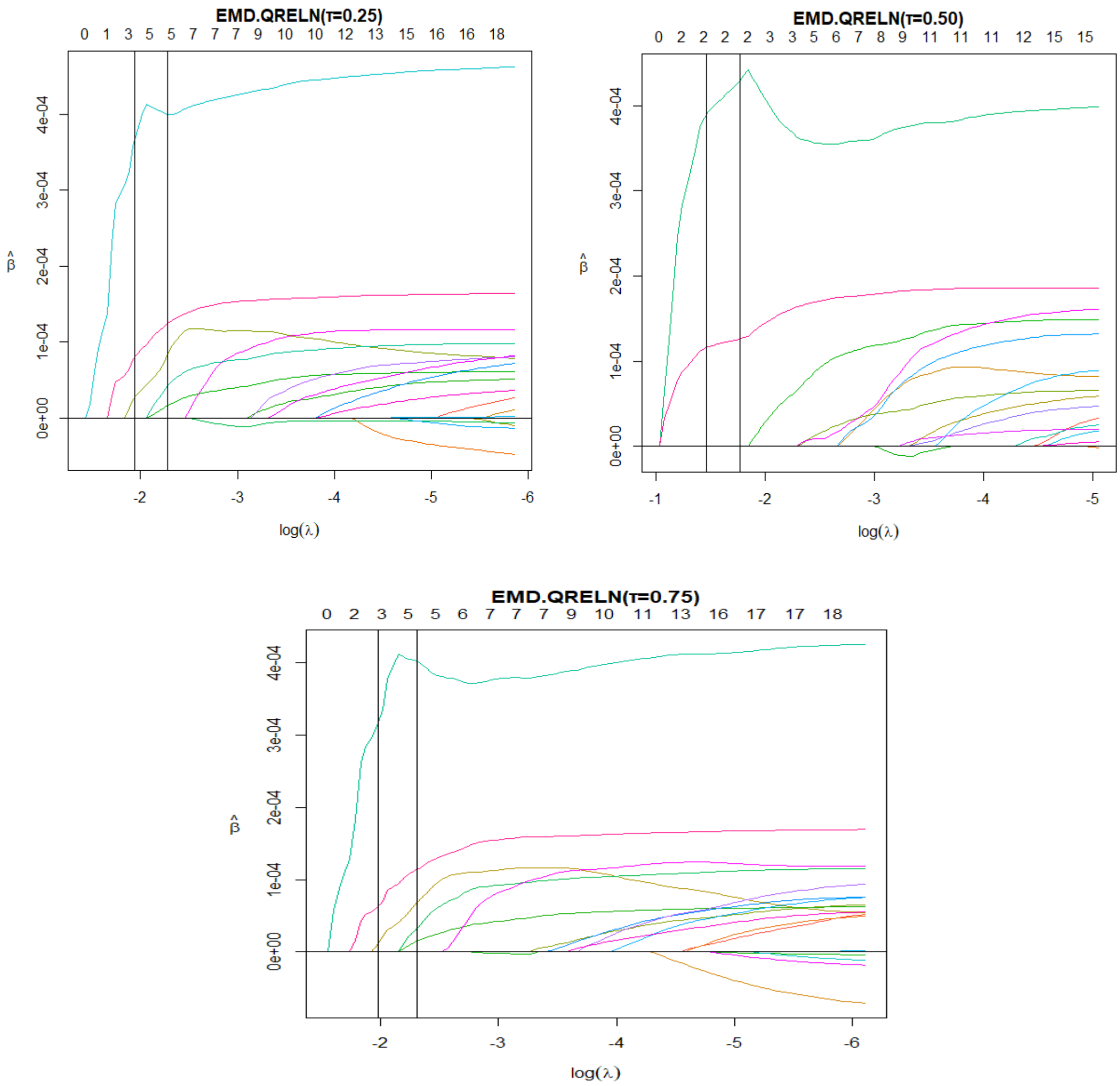

Figure 5. Coefficient estimates for the EMD.QRELN α_{opt} using a 10-CV.

(Table 3) outlines the performance criteria used to evaluate the prediction accuracy of the proposed method in comparison to existing methods. This comparison utilizes RMSE, MAE, MASE, MAPE, and EP as measures of prediction accuracy. The proposed $\mathrm{EMD.QRELN}_{\alpha_{Opt}}$ method provides the smallest error values across all three cases (τ = 0.25, 0.5, 0.75) for RMSE, MAE, and MASE. For example, at τ = 0.25, λ_{min} = 0.0138165; at τ = 0.5, λ_{min} = 0.01843012; and at τ = 0.75, λ_{min} = 0.028021. The method achieves the smallest error values. While MAPE shows a different order at τ = 0.25 and 0.5, and MAE shows a second-best order at τ = 0.75, ELNET.QR αopt generally improves prediction accuracy by minimizing error values in RSS, RMSE, MAE, and MAPE.

Table 3: Mean performance criteria.

Methods	λ	EP	RMSE	MASE	MAE	MAPE
			$\tau = 0.25$			
EMD. QRRidge	λ_{min}=0.1354017	0.0735	0.55123	3.45939	0.36182	**0.81407**
	λ_{1se}=0.1438492	0.07918	0.5864	3.66863	0.38371	0.83219
EMD. QRLasso	λ_{min}=0.01354017	0.06305	0.48517	3.08717	0.32289	0.89202
	λ_{1se}=0.01673463	0.06503	0.49939	3.17824	0.33242	0.89774
EMD.QRELN $\alpha = 0.25$	λ_{min}=0.05416067	0.06444	0.49474	3.12984	0.32735	0.86546
	λ_{1se}=0.01607335	0.06762	0.51287	3.2439	0.33928	0.85952
EMD.QRELN $\alpha = 0.50$	λ_{min}=0.02708033	0.06328	0.48793	3.09617	0.32383	0.88119
	λ_{1se}=0.03247165	0.06572	0.50348	3.19121	0.33377	0.87424
EMD.QRELN $\alpha = 0.75$	λ_{min}=0.01805356	0.06313	0.48609	3.09007	0.32319	0.88821

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	λ_{1se}=0.02100252	0.06668	0.50768	3.23313	0.33816	0.89992
EMD.QRELN $\alpha_{opt}=0.96$	λ_{min}=0.0138165	0.06304	0.48515	3.08711	0.32288	0.89141
	λ_{1se}=0.06494329	0.06534	0.50097	3.18821	0.33344	0.89682
			$\tau = 0.50$			
EMD. QRRidge	λ_{min}=0.1695571	0.06801	0.52125	3.40035	0.34846	0.84132
	λ_{1se}=0.2294738	0.08317	0.57054	3.79436	0.38884	0. 82041
EMD. QRLasso	λ_{min}=0.01695571	0.0244	0.40836	2.74748	0.28156	1.14219
	λ_{1se}=0.03723923	0.03269	0.42369	2.88446	0.2956	1.17641
EMD.QRELN $\alpha=0.25$	λ_{min}=0.06782285	0.03961	0.43336	2.85506	0.29258	1.03605
	λ_{1se}=0.03927092	0.04498	0.44516	2.93077	0.30034	1.01776
EMD.QRELN $\alpha=0.50$	λ_{min}=0.03391142	0.02673	0.41285	2.74837	0.28165	1.10681
	λ_{1se}=0.05846513	0.03499	0.42615	2.8485	0.29191	1.10151
EMD.QRELN $\alpha=0.75$	λ_{min}=0.02260762	0.02532	0.40987	2.74696	0.28159	1.13113
	λ_{1se}=0.04534342	0.03486	0.42972	2.93003	0.30027	1.18098
EMD.QRELN, $\alpha_{opt}=0.92$	λ_{min}=0.01843012	0.02464	0.40816	2.74119	0.28153	0.73915
	λ_{1se}=0.09178954	0.03283	0.42359	2.8771	0.29484	1.16689
			$\tau = 0.75$			
EMD. QRRidge	λ_{min}=0.168767	0.17635	0.56355	4.58799	0.45898	1.09035
	λ_{1se}=0.2354216	0.24466	0.66383	5.64775	0.565	1.30945
EMD. QRLasso	λ_{min}=0.02577932	0.11629	0.42697	3.33352	0.33348	1.08985
	λ_{1se}=0.03937817	0.11899	0.43157	3.39963	0.3401	1.11112
EMD.QRELN $\alpha=0.25$	λ_{min}=0.06750679	0.11071	0.43413	3.34309	0.33444	0.98719
	λ_{1se}=0.1062853	0.12863	0.4588	3.59212	0.35935	0.97209
EMD.QRELN $\alpha=0.50$	λ_{min}=0.04299829	0.11357	0.42776	3.31359	0.33149	1.04772
	λ_{1se}=0.06769818	0.11637	0.43459	3.39245	0.33938	1.03633
EMD.QRELN $\alpha=0.75$	λ_{min}=0.0333479	0.11476	0.42703	3.3251	0.33264	1.07069
	λ_{1se}=0.05093925	0.12634	0.4423	3.50261	0.3504	1.11158
EMD.QRELN $\alpha_{opt}=0.92$	λ_{min}=0.028021	0.11035	0.42701	3.32218	0.33035	1.03423
	λ_{1se}=0.04280236	0.11882	0.43204	3.4005	0.34019	1.10142

Table 4 shows the results of the bias, RSS, number of variable selections, and values of the tuning parameter λ of the proposed method and the previous methods. Based on RSS values, the EMD.QRELN method achieved the lowest RSS among all methods tested with αopt = 0.96: at τ = 0.25 (λ_{min} = 0.0138165; RSS = 332.1661; V.S = 6) and τ = 0.5 (λ_{min} = 0.01843012; RSS = 96.07629; V.S = 10). However, at τ = 0.75 (λ_{min} = 0.028021; RSS = 105.8748; V.S = 9), its RSS value was the smallest.

According to the findings in the preceding section, we will utilize the ELNET.QR αopt calculated coefficients to elucidate the final model as illustrated in (Table 4), as it demonstrates greater consistency in RMSE, MAE, MAPE, and MASE metrics. In conclusion, increased stationary time series data enhances prediction performance, motivating efforts to improve data stationarity for greater accuracy. Accuracy significantly improves when EMD multi-scale data decomposition is used across all methods. Hybrid methods incorporating EMD outperform existing methods, yielding models with reduced multicollinearity, heterogeneity, and prediction error.

Table 4: Coefficients estimation for the predictor variables and RSS error values.

Methods	λ	**Bias**	**RSS**	**V.S**
		$\tau = 0.25$		
EMD.RidgeQR	λ_{min}=0.1354017	0.103595	380.1965	$x_1, x_2, \dots x_{15}$
	λ_{1se}=0.1438492	0.107133	382.3293	$x_1, x_2, \dots x_{15}$
EMD.LassoQR	λ_{min}=0.01354017	0.026606	333.1223	x_2, x_3, x_4, x_8, x_9
	λ_{1se}=0.01673463	0.027609	332.7955	x_2, x_3, x_4, x_9
ElastcNetQR $\alpha=0.25$	λ_{min}=0.05416067	0.031945	337.3951	$x_1, x_2, .. x_{10}, x_{12} \dots x_{15}$
	λ_{1se}=0.01607335	0.034583	336.032	x_2, x_3, x_4, x_9
ElastcNetQR $\alpha=0.50$	λ_{min}=0.02708033	0.029219	336.0339	$x_1, .. x_5, x_7 \dots x_{10}$
	λ_{1se}=0.03247165	0.031175	336.7401	x_2, x_3, x_4, x_9
ElastcNetQR $\alpha=0.75$	λ_{min}=0.01805356	0.027283	332.7136	$x_1, .. x_5, x_7 \dots x_9$
	λ_{1se}=0.02100252	0.029897	346.8505	x_2, x_3, x_4, x_9

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ElastcNetQR , $\alpha_{opt} = 0.96$	λ_{min}=0.0138165	0.02652	332.1661	$x_2, x_3, x_4, x_5, x_8, x_9$
	λ_{1se}=0.06494329	0.027355	332.9736	x_2, x_3, x_4, x_9
		$\tau = 0.50$		
EMD.RidgeQR	λ_{min}=0.1695571	0.018503	157.8592	$x_1, x_2, \dots x_{15}$
	λ_{1se}=0.2294738	0.02767	189.1258	$x_1, x_2, \dots x_{15}$
EMD.LassoQR	λ_{min}=0.01695571	0.002381	96.88562	$x_1, \dots x_8, x_{10} \dots x_{12}, x_{14}$
	λ_{1se}=0.03723923	0.004273	104.2974	x_3, x_4, x_{10}, x_{11}
ElastcNetQR $\alpha = 0.25$	λ_{min}=0.06782285	0.006275	109.1128	$x_1, \dots x_8, x_{10} \dots, x_{14}$
	λ_{1se}=0.03927092	0.004862	115.1375	x_3, x_4, x_{10}, x_{11}
ElastcNetQR $\alpha = 0.50$	λ_{min}=0.03391142	0.002859	99.02917	$x_1, \dots x_8, x_{10} \dots x_{12}, x_{14}$
	λ_{1se}=0.05846513	0.008093	105.5105	x_3, x_4, x_{10}, x_{11}
ElastcNetQR $\alpha = 0.75$	λ_{min}=0.02260762	0.002565	97.71988	$x_1, \dots x_8, x_{10} \dots x_{12}, x_{14}$
	λ_{1se}=0.04534342	0.004897	107.2894	x_3, x_4, x_{10}, x_{11}
ElastcNetQR , $\alpha_{opt} = 0.92$	λ_{min}=0.01843012	0.002428	96.07629	$x_1, \dots x_6, x_8, x_{10}, \dots x_{12}$
	λ_{1se}=0.09178954	0.004313	104.2471	x_3, x_4, x_{10}, x_{11}
		$\tau = 0.75$		
EMD.RidgeQR	λ_{min}=0.168767	0.05529	184.5164	$x_1, x_2, \dots x_{15}$
	λ_{1se}=0.2354216	0.106413	256.0331	$x_1, x_2, \dots x_{15}$
EMD.LassoQR	λ_{min}=0.02577932	0.024043	105.9191	x_2, x_3, x_4
	λ_{1se}=0.03937817	0.025172	108.2116	$x_2, \dots, x_4$
ElastcNetQR $\alpha = 0.25$	λ_{min}=0.06750679	0.02179	109.5	$x_2, \dots x_5, x_7, x_{9,\dots} x_{13}, x_{15}$
	λ_{1se}=0.1062853	0.029412	122.2996	x_2, x_3, x_4
ElastcNetQR $\alpha = 0.50$	λ_{min}=0.04299829	0.022932	106.3106	$x_2, \dots x_5, x_7, \dots, x_9, x_{12} \dots x_{15}$
	λ_{1se}=0.06769818	0.024076	109.7326	x_2, x_3, x_4
ElastcNetQR $\alpha = 0.75$	λ_{min}=0.0333479	0.023412	106.1299	$x_2, \dots x_5, x_{7,} \dots, x_9, x_{13} \dots x_{15}$
	λ_{1se}=0.05093925	0.028379	113.6597	x_2, x_3, x_4
ElastcNetQR , $\alpha_{opt} = 0.92$	λ_{min}=0.028021	0.023901	105.8748	$x_2, \dots x_4, x_7, \dots x_9, x_{13} \dots x_{15}$
	λ_{1se}=0.04280236	0.025092	108.4483	x_2, x_3, x_4

Conclusion

This study introduces a hybrid EMD-QRELN method that uses non-stationary and nonlinear predictor variables to identify which components from the EMD of the original predictors have the greatest effect on the response variable. The EMD. The QRELN method is predicated on the selection of the optimal alpha value α_{Opt} through a cross-validation approach. This approach is used to identify relationships between predictor variables and response variables to enhance model selection accuracy and address issues such as heavy-tailed distributions, heterogeneity, and multicollinearity in predictor variables by selecting the optimal alpha value. The results of the numerical experiments and stock market applications prove that the $EMD.QRELN_{\alpha_{Opt}}$ is highly capable of identifying predictor variables that have the most significance on the response variable, resulting in reduced prediction errors at τ = 0.25, 0.5, and 0.75. Furthermore, it demonstrated that not all alpha values are suitable for Elastic Net, making cross-validation the preferred method for selecting the optimal alpha value for the final model.

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